1. Any ray matrix \( \begin{bmatrix} A & B \\ C & D \end{bmatrix} \) has
\[ AD - BC = \frac{n_{in}}{n_{out}} \]

For symmetric system, \( n_{in} = n_{out} \)
So \( AD - BC = 1 \)

This rules out (b): \( AD - BC = 4 \)
and (d): \( AD - BC = -1 \)

Easiest approach: think of a few symmetric systems:

- Single thin lens \( M = \begin{bmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{bmatrix} \)

- Free propagation distance \( d \): \( M = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \)

Both of these have \( A = D \), so guess \( (a) \) with confidence

To make sure:

If input ray \( \hat{u}_1 \) leads to output \( \hat{u}_2 \),
then input \( \hat{u}_2' = \begin{bmatrix} \theta_2 \\ -\theta_2 \end{bmatrix} \) leads to output \( \hat{u}_1' = \begin{bmatrix} \theta_1 \\ -\theta_1 \end{bmatrix} \)

for a symmetric system:
Try \( u_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \)

Then for (a), \( \tilde{u}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \) \( \tilde{u}_2' = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \)

and \( M_a \tilde{u}_2' = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \tilde{u}_1' \)

But for (a), \( \tilde{u}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) \( \tilde{u}_2' = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \)

and \( M_a \tilde{u}_2' = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \neq \tilde{u}_1' \)

So answer must be \( (a) \)

**General Solution:**

\[
\begin{align*}
u_1 &= \begin{bmatrix} y_0 \\ \theta \end{bmatrix} \\
u_2 &= \begin{bmatrix} A y_0 + B \theta \\ C y_0 + D \theta \end{bmatrix} \\
u'_2 &= \begin{bmatrix} A y_0 + B \theta \\ -C y_0 - D \theta \end{bmatrix}
\end{align*}
\]

\[
M\tilde{u}_2' = \begin{bmatrix} A^2 y_0 + AB \theta - BC y_0 - BD \theta \\ AC y_0 + BC \theta - DC y_0 - D^2 \theta \end{bmatrix} = \begin{bmatrix} y_0 \\ -\theta \end{bmatrix}
\]

Need

\[
\begin{align*}
A^2 - BC &= 1 \\
AB - BD &= 0 \\
AC - DC &= 0 \\
D^2 - BC &= 1
\end{align*}
\]

Then \( AD - BC = A^2 - BC = D^2 - BC = 1 \) anyway.
2. Plane wave is defined by
\[ U(\vec{r}) = A e^{-i k_0 \vec{r}} \]
\[ u(\vec{r}) = \text{Re}(U(\vec{r}) e^{i\omega t}) \]
\[ = |A| \cos(\omega t - k_0 \cdot \vec{r} + \phi) \]

So:
(a) Wave, but not plane wave
(b) Plane wave with \( \phi = -\frac{\pi}{2} \)
(c) Plane wave with \( A = 1, \ k = 0 \)
(d) Not a plane wave
(e) Looks like plane wave with \( \vec{k} = \left( \frac{-\pi}{\lambda}, \frac{-\pi}{\lambda}, \frac{\sqrt{2} \pi}{\lambda} \right) \)

check \( \vec{k}^2 = \frac{\pi^2}{\lambda^2} \) \( 1 + 1 + 2 = \frac{2 \pi^2}{\lambda^2} \)

so it is a good solution.

3. Say red slit generates diffracted wave \( U_R(t) \)
blue slit \( U_B(t) \)

Then interference term is \( \langle U_R^* U_B \rangle \)

But if \( U_R \) oscillates near \( U_R \)
and \( U_B \) oscillates near \( U_B \)
with \( \omega_0 - \omega \) large,

then time average
\[ \langle U_R^* U_B \rangle \sim \langle e^{-i(\omega_R - \omega_B) t} \rangle \to 0 \]

So no interference observed anywhere, \( (a) \)
4. Wave propagates as \( e^{i(\omega t - kz)} \)
   
   \[ e^{\frac{\alpha^2}{2} i(\omega t - nk_0 z)} \]
   
   \( n = \text{Re} \, \tilde{n} \)

   Phase velocity is \( c = \frac{\omega}{nk_0} = \frac{c_0}{n} \)

   \[ \frac{c_0}{1.5} = 0.667 \, c_0 \]

5. Label pictures (a) (b) (c) (d)

   (Sorry I forgot labels on exam!)

   Can rule out (a) \& (b) because center of Fraunhofer pattern is always bright

   (Recall problem 6 from midterm.)

   To judge between (c) \& (d), note fastest spatial frequency in horizontal direction is \( \frac{1}{a} \)

   while in vertical direction its \( \frac{2}{a} \)

   Since diffraction pattern \( F(\omega_x = \frac{x}{2a}, \omega_y = \frac{2y}{3a}) \)

   expect large \( y \) features to have twice the length scale of large \( x \) features.

   Look at central big rectangle: in (c), sides have 2:1 ratio. In (d), sides have 3:1 ratio.

   So, choose [C]
How I made pictures:

Can write $\begin{array}{c}
\Huge + \\
\Huge \text{as} \\
\Huge \begin{array}{c}
\Box \\
\Box - \Box
\end{array}
\end{array}$

Since everything is linear, get

$$U(x,y) = \text{rect}(a, 3a) + \text{rect}(3a, \frac{3a}{2}) - \text{rect}(a, \frac{a}{2})$$

where $\text{rect}(Dx, Dy)$ is pattern from rectangular slit

$$\text{rect}(Dx, Dy) = \frac{Dx \cdot Dy}{\lambda d} \cdot \text{sinc} \left( \frac{x \cdot D_x}{\lambda d} \right) \cdot \text{sinc} \left( \frac{y \cdot D_y}{\lambda d} \right)$$

6. Here TE is normal to page, and optic axis is in page, so TE is ordinary, TM is extraordinary.

If $n_0 > n_1$, then $n_{TE} < n_{TM}$

So TE light refracts less:

$$\sin \Theta_2 = \frac{n_1}{n_2} \sin \Theta_1$$

the closer $n_1$ and $n_2$ are, the closer $\Theta_1$ and $\Theta_2$ are.

So, choose $(b)$
Multiple Choice Scores:

% correct:
1. 15%
2. 3.7%
3. 15%
4. 70%
5. 41%
6. 74%

Average score: 11.3/30
7. \( U(z, 0) = A \left[ \frac{1}{4} (e^{\frac{4\pi i}{\lambda} (z-b)} + e^{\frac{4\pi i}{\lambda} (z-b)} + e^{-\frac{4\pi i}{\lambda} (x-b)} + e^{-\frac{4\pi i}{\lambda} (x-b)}) - 1 \right] \)

So \( v_x, v_y = \pm \frac{1}{2\lambda} \) for first terms

\( = 0 \) for last term

Then \( v_z = \sqrt{v_x^2 + v_y^2} = \frac{1}{\sqrt{2\lambda}} \) for first terms

\( v_z = \frac{1}{\lambda} \) for last term

So \( U(x, y, z) = A \left[ e^{-\frac{2\pi i z}{\lambda}} \cos \frac{\pi x}{\lambda} \cos \frac{\pi y}{\lambda} - e^{-\frac{2\pi i z}{\lambda}} \right] \)

\( U(0, 0, z) = A \left[ e^{-\frac{2\pi i z}{\lambda}} - e^{-\frac{2\pi i z}{\lambda}} \right] = 0 \)

Need \( \frac{2\pi n}{\sqrt{2\lambda}} = 2\pi n + \frac{2\pi n}{\lambda} \) for integer \( n \)

\( \sqrt{2}(\frac{1}{\sqrt{2}} - 1) = n \lambda \)

\( z = n \frac{\lambda}{\sqrt{2} - 1} \) or \( \frac{2\lambda}{n \sqrt{1 - \frac{1}{\sqrt{2}}} \} \)
8. From Fresnel equations,

\[ r_+ = 1 \quad \text{if} \quad n_2 < \infty \]
\[ r_- = +1 \]

From Figure 6.2-1 see that in either case

get \( E_{\text{ref}} = -E_{\text{in}} \) at normal incidence

So \( E_{\text{tot}} = E_{\text{in}} - E_{\text{ref}} \)

\[ E_{\text{tot}} = A \hat{z} e^{-ikz} - A \hat{z} e^{+ikz} \]

\[ E_{\text{tot}} = -2iA \hat{x} \sin k\hat{z} \]

Then \( \hat{H} = \hat{H}_{\text{in}} - \hat{H}_{\text{ref}} \)

\[ H_{\text{in}} = \frac{i}{\eta_0} k x E_{\text{in}} = \frac{i}{\eta_0} (\hat{x} \times \hat{z}) A e^{-ikz} \]

\[ = \frac{1}{\eta_0} \hat{y} A e^{-ikz} \]

while \( \hat{H}_{\text{ref}} = \frac{i}{\eta_0} k_{\text{ref}} \times E_{\text{ref}} = \frac{i}{\eta_0} (-\hat{x} \times \hat{z}) A e^{ikz} \]

\[ = \frac{1}{\eta_0} \hat{y} A e^{ikz} \]

So \( \hat{H}_{\text{tot}} = \frac{A}{\eta_0} \hat{y} (e^{-ikz} - e^{ikz}) = \frac{2A}{\eta_0} \hat{y} \cos k\hat{z} \)

Above for \( z < 0 \). For \( z > 0 \), have \( E, H \sim e^{\frac{\alpha z^2}{2}} \rightarrow 0 \) within conductor.
q. Say initial polarization is \( \vec{J}_{in} = \begin{bmatrix} J_x \\ J_y \end{bmatrix} \)

write as 
\[
\vec{J}_{in} = \frac{1}{2} \begin{bmatrix} J_x \\ J_y \end{bmatrix} + \frac{J_x}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{J_y}{2i} \begin{bmatrix} 1 \\ i \end{bmatrix} - \frac{J_y}{2i} \begin{bmatrix} 1 \\ -i \end{bmatrix} 
\]

\[
= \frac{1}{2} \begin{bmatrix} J_x (t_{R} + t_{L}) + \frac{1}{i} J_y (t_{R} t_{L} - t_{L} t_{R}) \end{bmatrix} 
\]

Pass through medium:
\[
\vec{J}_{out} = \frac{1}{2} \begin{bmatrix} J_x (t_{R} t_{L} - t_{R} + t_{L}) + \frac{1}{i} J_y (t_{R} t_{L} - t_{L} t_{R}) \end{bmatrix} 
\]

\[
= \frac{1}{2} \begin{bmatrix} J_x (t_{R} + t_{L}) - i \frac{1}{i} J_y (t_{R} - t_{L}) \\ J_x (t_{R} - t_{L}) + J_y (t_{R} + t_{L}) \end{bmatrix} 
\]

So
\[
\vec{J}_{out} = \frac{1}{2} \begin{bmatrix} t_{R} + t_{L} & -i (t_{R} - t_{L}) \\ i (t_{R} - t_{L}) & t_{R} + t_{L} \end{bmatrix} \vec{J}_{in} 
\]

Could also write \( M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \) and solve \( M \begin{bmatrix} \vec{1} \\ \vec{i} \end{bmatrix} = t_{R} \begin{bmatrix} \vec{1} \\ \vec{i} \end{bmatrix} \)

\[
M \begin{bmatrix} \vec{1} \\ \vec{i} \end{bmatrix} = t_{L} \begin{bmatrix} \vec{1} \\ \vec{-i} \end{bmatrix} 
\]

Or, write \( M = U^{-1} \begin{bmatrix} t_{R} & 0 \\ 0 & t_{L} \end{bmatrix} U \) for \( U = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \)
10. Evaluate

\[ G(\tau) = \langle U^*(t) U(t+\tau) \rangle \]

\[ U(t) = \frac{1}{\sqrt{2}} [U_0(t) + U_0(t-a)] \]

\[ G(\tau) = \frac{i}{2} \langle [U_0^*(t) U_0(t+\tau)] [U_0(t+\tau) + U_0(t-a+\tau)] \rangle \]

\[ = \frac{i}{2} \left[ \langle U_0^*(t) U_0(t+\tau) \rangle + \langle U_0^*(t-a) U_0(t+\tau) \rangle \\
+ \langle U_0^*(t) U_0(t-a+\tau) \rangle + \langle U_0^*(t-a) U_0(t-a+\tau) \rangle \right] \]

\[ = \frac{i}{2} \left[ G_0(\tau) + G_0(\tau+a) + G_0(\tau-a) + G_0(\tau) \right] \]

where \( G_0(\tau) = \langle U_0^*(t) U_0(t+\tau) \rangle \)

Then

\[ S(\nu) = \int_{-\infty}^{\infty} e^{-2\pi i \nu \tau} G(\tau) \, d\tau \]

\[ = S_0(\nu) + \frac{i}{2} \int_{-\infty}^{\infty} e^{-2\pi i \nu \tau} [G_0(\tau+a) + G_0(\tau-a)] \, d\tau \]

\[ = S_0(\nu) + \frac{i}{2} \int_{-\infty}^{\infty} [e^{-2\pi i \nu (\tau-a)} + e^{-2\pi i \nu (\tau+a)}] G_0(\tau) \, d\tau \]

\[ = S_0(\nu) + \frac{i}{2} \left( e^{2\pi i \nu a} + e^{-2\pi i \nu a} \right) S_0(\nu) \]

\[ S(\nu) = S_0(\nu) \left[ 1 + \cos(2\pi \nu a) \right] \]