Phys 531 Assignment 1 Due September 14

1. For each of the following complex number expressions, find the real part, the imaginary part, the magnitude, and the phase. You should do the complex number manipulations by hand, but you can use a calculator for evaluating algebraic expressions or trig functions if you wish. For (a) and (f), express your results in terms of the real parameters a and b.

(a)

$$\frac{1}{a+ib}$$

(b)

$$\frac{1+i}{2+i}$$

(c)

$$(3-2i)\exp\left(\frac{3i\pi}{4}\right)$$

(d)

$$\exp\left(\frac{i\pi}{3}\right)\cdot\exp\left(\frac{i\pi}{4}\right)$$

(e)

$$(1+i)^{3/2}$$

(f)

$$\cos(a+ib)$$

Hint: for (f), use $\cos\alpha = \frac{1}{2}(e^{i\alpha}+e^{-i\alpha})$.

2. Determine which of the following functions is a solution of the wave equation. For those that are solutions, find the wave speed v in terms of the parameter c or ω and k.

(a)

$$\psi(z,t) = \frac{z}{ct}$$

(b)

$$\psi(z,t) = z^2 + c^2t^2$$

(c)

$$\psi(z,t) = e^{-(z^2+c^2t^2)}$$

(d)

$$\psi(z,t) = \sin 2kz \sin\omega t$$

(e)

$$\psi(\mathbf{r},t) = \frac{1}{x+y+2z-ct}$$

(f)

$$\psi(\mathbf{r},t) = e^{i(\omega t+ky)} + e^{i(\omega t-kz)}$$

(Of these, only (d) and (f) are physically interesting; the rest are meant to help you gain familiarity with how the wave equation works.)

3. A wave is travelling at speed $c = 3 \times 10^8$ m/s. Calculate the wave number k (in m^{-1}) and the wave frequency in cyles per second (ν) and radians per second (ω) for wavelengths λ of 500 nm, 10.6 μm, 0.6 mm, and 10 cm.

4. Suppose a point source emits a spherical wave with $\lambda = 600$ nm. How far from it must we be so that the resulting optical field deviates from a plane wave by less than one-eighth of a wavelength over an illuminated spot 1 cm in radius?

5. In a medium with an electric current density $\mathbf{J}$, Maxwell's equation (3.19) is modified to be $\nabla \times \mathbf{B} = \mu\,(\mathbf{J} + \epsilon \partial \mathbf{E}/\partial t)$, with the other equations unaltered. If the medium is described by Ohm's law, then $\mathbf{J} = \sigma \mathbf{E}$, where σ is the conductivity. In this case, show that plane waves of the form

$$\mathbf{E} = \mathbf{E}_0 e^{i(\omega t - kx)}$$

are still solutions, but with a complex-valued k. Give an explicit expression for k in terms of ω, ϵ, μ_0, and σ.

6. The average irradiance of sunlight at the earth's surface is 250 W/m^2. Consider a monochromatic plane wave in vacuum with this irradiance and wavelength 550 nm. Calculate the amplitudes of
(a) the electric field (E_0)
(b) the magnetic flux density (B_0), and
(c) the time-averaged energy density.
Your answers should have the correct units.

7. Pulses of UV light lasting 5.0 ns each are emitted from an excimer laser that has a beam of diameter 2.5 mm. If each burst carries an energy of 3.0 mJ, (a) determine the length in space of each pulse, and (b) find the energy per unit volume, the irradiance, and the electric field amplitude in the pulses.