Phys 531 Assignment 10 Due December 7

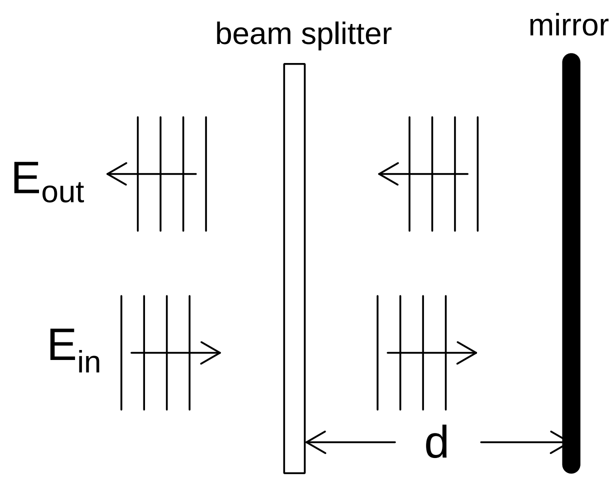
1. In general, a lossless beam splitter can be characterized by its complex transmittance $t$ and complex reflectance $r$. In other words, when a wave $E_{\rm in}$ is incident on the beam splitter, a wave $rE_{\rm in}$ is reflected from the front surface and a wave $tE_{\rm in}$ is transmitted out the rear surface. Conservation of energy evidently requires that $|r|^2 + |t|^2 = 1$. You may use this result below, and you may also assume that the front and back faces of the beam splitter are symmetric.

A less obvious relation between $r$ and $t$ can be obtained by considering the peculiar interferometer shown below, in which the beam splitter is placed a distance $d$ in front of a perfectly reflecting plane mirror. The incident light is a plane wave at normal incidence, and the light acquires a phase of $\pi$ when it bounces off the mirror.
(a) By summing the multiple reflected fields, find an expression for the total wave reflected by the interferometer.
(b) Since there is only one output beam and energy is conserved, the irradiance of the reflected wave must equal the irradiance of the incident wave, regardless of the spacing $d$. Using this fact, show that if $r = |r|e^{i\phi}$, then $t$ must satisfy

$$t = \pm i|t|e^{i\phi}$$

Thus the transmitted and reflected fields are always 90° out of phase.

Hint: if an equation of the form $A + Be^{i\phi} + Ce^{-i\phi} = 0$ holds for all $\phi$, then $A$, $B$ and $C$ must all be zero.




2. Suppose a Gaussian laser beam with $\lambda = 532$ nm is collimated with a width of 0.5 mm. If the light is passed through a thin lens, what focal length is required to produce a new focus at a distance of 50 cm? What is the beam waist at that focus? There are two possible solutions to this problem; you should find both.

3. Consider a deterministic wave with time dependence $E(t) = E_0 \cos(\Omega t) e^{-i\omega_0 t}$. Calculate:
(a) the Fourier transform $\mathcal{E}(\omega)$
(b) the correlation function $\Gamma(\tau)$ and the power spectral density $S(\omega)$
(c) the average irradiance of the wave.

4. Suppose you attempted to observe a two-slit interference pattern using sunlight as your source. You use a filter to obtain green light with $\lambda = 550$ nm and $\Delta\lambda = 25$ nm. The sun subtends an angle of about $0.5°$.
(a) What is the coherence time and the lateral coherence length for this source?
(b) Estimate the maximum slit separation $a$ for which the interference pattern will be apparent.
(c) If the slits are separated by this distance $a$, estimate the number of interference fringes that will be observed.

5. A point source at the coordinate origin $(0,0,0)$ emits light with a Lorentzian spectrum

$$S(\omega) = I_0 \frac{2\tau_c}{\tau_c^2(\omega - \omega_0)^2 + 1}$$

for coherence time $\tau_c$. Calculate the complex degree of spatial coherence $\gamma_{12}(0)$ for two points located at $\mathbf{r}_1 = (0,0,d)$ and $\mathbf{r}_2 = (x,0,d)$ with $d \gg x$. Sketch the magnitude of $\gamma_{12}$ as a function of $x$ for $d = 10$ cm and $\tau_c = 10$ ps.