Phys 531 Assignment 2   Due September 21

1. A light wave travels from point A to point B in vacuum, covering a distance of 1 cm. Suppose we introduce into its path a flat glass plate ($n_g = 1.50$) of thickness $L = 1.00$ mm. If the vacuum wavelength is 500 nm, how many wavelengths span the space from A to B with and without the glass in place? What phase shift is introduced with the insertion of the plate?

2. Suppose a laser produces a pulse of light with a duration of 1 ns, a diameter of 1 cm, and a total energy of 1 mJ. In free space, the pulse length is thus 30 cm, the energy density is 42 J/m^3, and the irradiance is 1.3×10^{10} W/m^2. Calculate the length, energy density and irradiance if the pulse is instead in a medium with $n = 1.5$. How do the electric fields of the two cases compare?

3. The optics catalog from the CVI Laser Corporation gives the index of refraction for fused silica (a type of glass) as a function of (vacuum) wavelength λ:

$$n^2(\lambda) = 1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3}$$

with $B_1 = 0.696$, $B_2 = 0.407$, $B_3 = 0.897$, $C_1 = 4.679 \times 10^{-3}$ μm^2, $C_2 = 1.351 \times 10^{-2}$ μm^2, and $C_3 = 9.793 \times 10^1$ μm^2. On the basis of this formula, over what range of wavelengths might you expect fused silica to be transparent? (Note, this is a formula just for the real index, not the full complex index. I want you to figure out what the imaginary part must be doing.)

4. In class we discussed a simple model for an atom, in which a cloud of negative charge (the electrons) surrounds a fixed nucleus of positive charge. Develop the details of this model as follows: Suppose an atom has one electron with charge $-e$ and a nucleus of charge $+e = 1.6 \times 10^{-19}$ C, and the electronic charge is uniformly distributed over a sphere with the Bohr radius $a_0 = 4\pi\epsilon_0\hbar^2/(me^2) = 5.29 \times 10^{-11}$ m. If the electron cloud is displaced a distance x, the magnitude of the attractive force between the cloud center and the nucleus is given by

$$F_{\text{Coulomb}} = \frac{1}{4\pi\epsilon_0}\frac{eq}{x^2}$$

where q is the portion of the electronic charge contained within a sphere of radius x. (This is easy to prove using Gauss's Law.)
(a) Using this result, show that the restoring force is linear in x and calculate the resulting oscillation frequency ω_0.
(b) Numerically compare your results to the Lyman-α resonance of hydrogen, which has a wavelength of 122 nm.

5. Consider the scattered field produced by a sphere of diameter $L \gg \lambda$, measured at a small but nonzero angle θ from directly forward. We showed in class that the scattered field at $\mathbf{r}$ coming from a point $\mathbf{r}_j$ within the sphere is

$$\mathbf{E}_j(\mathbf{r}) = \frac{k^2 \chi_1 E_0}{4\pi d} e^{i[k(x_j + d_j) - \omega t]}$$

where x_j is the distance from the front of the sphere and $d_j = |\mathbf{r} - \mathbf{r}_j|$ as shown. In order to have a large total scattered field at $\mathbf{r}$, the phases $\phi_j = k(x_j + d_j)$ from all the different points $\mathbf{r}_j$ must be the same to within a radian or so. How large can θ be while meeting this criterion? Assume that the measurement point $\mathbf{r}$ is very far away from the sphere, and you can also reasonably expect that θ will be small compared to 1.
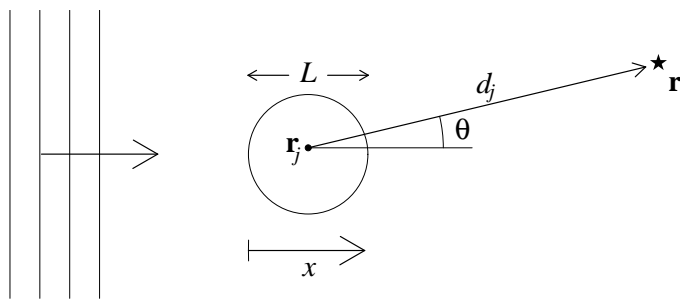
6. Consider the path taken by light emitted by a source at point P, bouncing off a spherical mirror of radius R, and detected at point P'. The point P' is at the center of curvature of the mirror and P is a distance z away. The figure shows one possible path the light could take. Use Fermat's principle to find the true path and show that the true path represents a maximum length if P is to the right of P' (as shown), but a minimum length if P is to the left of P'.
(Hint: you can determine whether a function has a maximum or minimum at a point using its second derivative.)
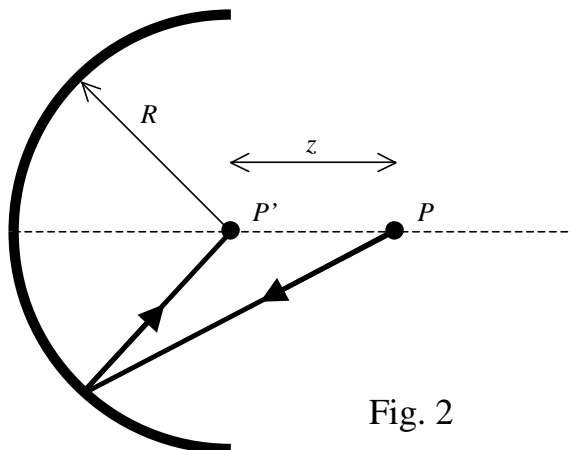

Fig. 2