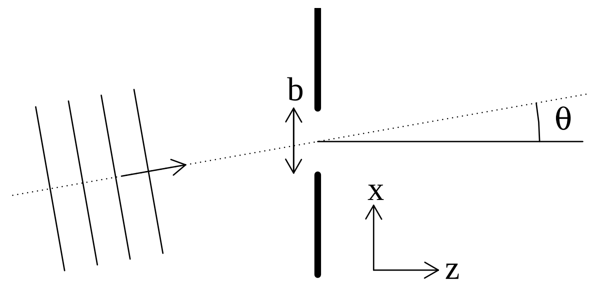
Phys 531 Assignment 8 Due November 9

1. Calculate the Fraunhofer diffraction pattern produced when a rectangular slit of width $b$ and height $L$ is illuminated by a plane wave making a small angle $\theta$ with respect to the plane of the slit, as shown. Show that the pattern itself propagates at the same angle $\theta$.




2. Consider an aperture consisting of two pinholes separated by a distance $2a$. Assume the pinholes are small enough to be approximated by $\delta$-functions, so that

$$A(x, y) = [\delta(x - a) + \delta(x + a)]\delta(y)$$

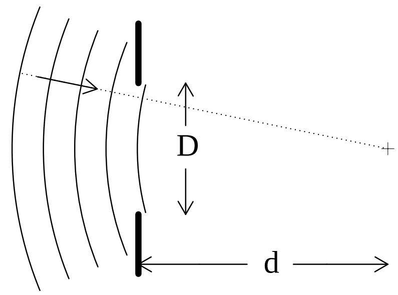
Calculate the diffracted field $A_d$ produced at a distance $d$, using
(a) the Fresnel approximation
(b) the Fraunhofer approximation

3. Suppose an aberration-free imaging system is used to observe two point sources at infinite object distance. The two resulting images are a distance $B$ apart. For large $B$, the points will be resolved and the image will show two distinct peaks. For small $B$, the points are not resolved and the image shows only one peak. In terms of the wavelength $\lambda$, the diameter of the lens $D$, and the focal length $f$, determine the value of $B$ separating these two regimes. Find an answer accurate to within 2%:
(a) Assuming the two points are mutually coherent. For instance, they might be two tiny holes in a screen illuminated by a plane wave.
(b) Assuming the two points are mutually incoherent. For instance, they might be two distant stars (observed through a filter that transmits only light of wavelength $\lambda$). This prevents the two waves from interfering, so the observed image will be the sum of the individual irradiances.
Hint: A straightforward way to solve this is to use a computer program capable of plotting Bessel functions and see visually how close the image centers can get before merging into a single peak.

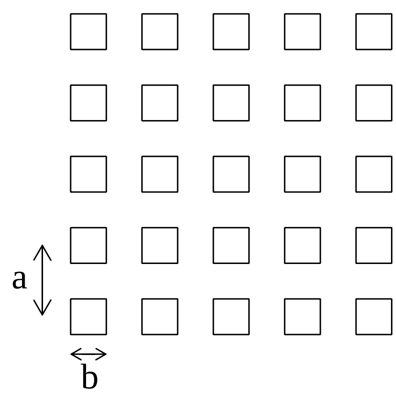
4. Suppose a converging spherical wave is incident on a circular aperture of diameter $D$, as shown. The field in the aperture is then

$$A(x,y) = E_0 \frac{e^{-ik\sqrt{x^2+y^2+d^2}}}{k\sqrt{x^2+y^2+d^2}}$$

Using the Fresnel approximation, calculate the resulting field $A_d(x,y)$ at $z=d$, the plane containing the center of the orginal wave. (Note that the converging wave might have been produced by a plane wave incident on a lens with $f=d$, so you should have a pretty good idea what answer to expect.)




5. (a) Calculate the Fraunhofer diffraction pattern produced by a plane wave normally incident on a screen consisting of square holes with side $b$ and center spacing $a$. There are a total of $N^2$ holes in the screen (in an $N \times N$ array).




(b) Suppose such a screen has $a = 0.5$ mm, $b = 0.1$ mm, and is illuminated with light having $\lambda = 500$ nm. The screen is the object of an imaging system as shown, where the lens focal length is $f = 100$ mm. A square hole with side $p$ is placed in the focal plane of the lens so as to low-pass filter the image. What value of $p$ should be used to give a smooth, uniform image with no modulation from the screen pattern? (You can assume here that $N \gg 1$. In fact, you can just take $N = 10$ to be concrete.)