Phys 531 Lecture 19 9 November 2004

Polarization of Light

Last time, finished Fourier optics

Saw lots of interesting applications

Next three lectures: polarization
explore vector nature of light

Today: basic ideas

No Fourier transforms required!

Outline:

- Notation and conventions
- Polarization states
- Basis states
- Unpolarized light
- Polarization and quantum mechanics

Follow book more closely again: Chapter 8

Note, book neglects complex notation until §8.13
- we'll use from beginning

Next time:
Generating and manipulating polarized light

Conventions

Have been ignoring vector nature of $\mathbf{E}$

- Not very important for diffraction
- Simplifies calculations

But it is important for many things

Already saw in Fresnel relations

When is polarization important?

When can we ignore polarization?

- Imaging problems
- Interference/diffraction for beams at small angles

When is it important?

- Transmittance/reflectance calcs
- Superposing beams at large angles
- Detailed interactions with matter:

Birefringent materials, surface effects, atomic/molecular transitions, nonlinear optics, magneto-optical effects, electro-optical effects, . . .

Beyond this course, but common applications

Review what we know:

Plane wave solution is

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

complex vector amplitude $\mathbf{E}_0$, know $\mathbf{k}\cdot\mathbf{E}_0 = 0$

Standard configuration: take $\mathbf{k} = k\hat{\mathbf{z}}$
Then $\mathbf{E}_0 = E_{0x}\hat{\mathbf{x}} + E_{0y}\hat{\mathbf{y}}$

Real fields are

$$E_x(z,t) = |E_{0x}| \cos(kz - \omega t + \phi_x)$$
$$E_y(z,t) = |E_{0y}| \cos(kz - \omega t + \phi_y)$$

Phases ϕ_x, ϕ_y are independent

Relation between phases sets polarization
- along with amplitudes $|E_{0x}|, |E_{0y}$

Define phase difference $\varepsilon = \phi_y - \phi_x$

Write

$$E_x(z,t) = |E_{0x}| \cos(kz - \omega t + \phi)$$
$$E_y(z,t) = |E_{0y}| \cos(kz - \omega t + \phi + \varepsilon)$$

don't worry about overall phase ϕ

In complex form:

$$E_x(z,t) = |E_{0x}| e^{i\phi} e^{i(kz-\omega t)}$$

$$E_y(z,t) = |E_{0y}| e^{i(\phi+\varepsilon)} e^{i(kz-\omega t)}$$

Define complex amplitude

$$E_0 = \sqrt{|E_{0x}|^2 + |E_{0y}|^2}\, e^{i\phi}$$

and polarization vector = Jones vector =

$$\hat{\jmath} = \frac{|E_{0x}|}{|E_0|}\,\hat{\mathbf{x}} + \frac{|E_{0y}|}{|E_0|} e^{i\varepsilon}\,\hat{\mathbf{y}}$$

Then $\mathbf{E}(z,t) = E_0 \hat{\jmath} e^{i(kz-\omega t)}$

Polarization States (Hecht 8.1)

Look at $\mathbf{E}$ for different ε

Take $\phi = 0$ for simplicity

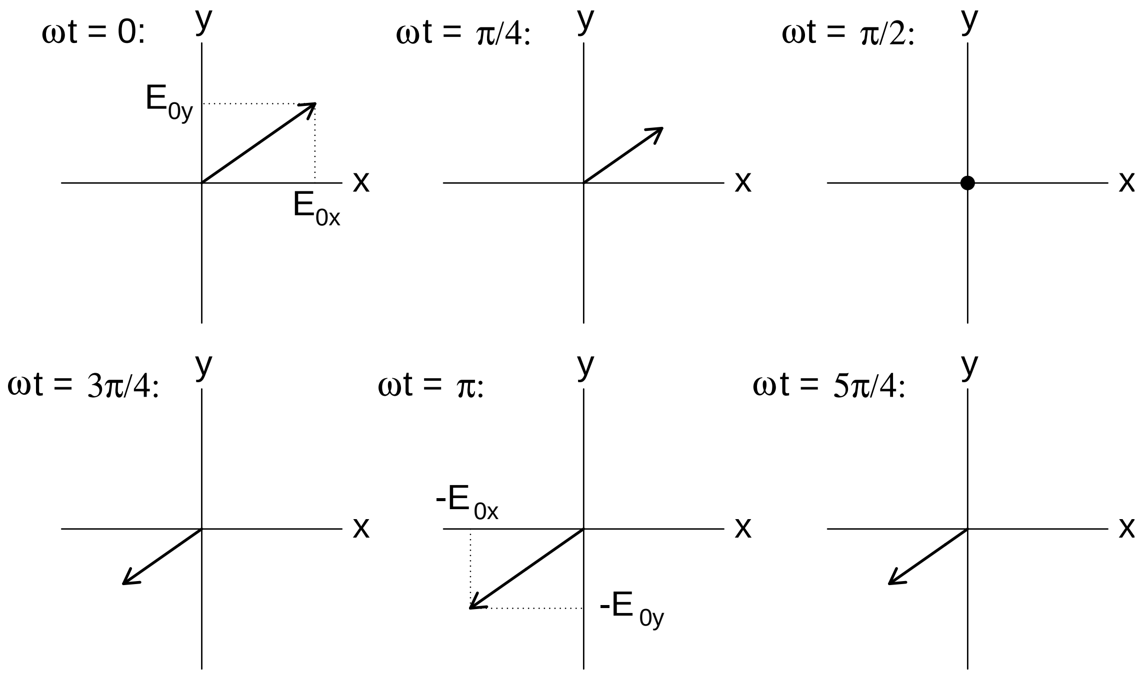
Suppose $\varepsilon = 0$

Then $E_x(z,t) = |E_{0x}| \cos(kz - \omega t)$

$E_y(z,t) = |E_{0y}| \cos(kz - \omega t)$

When E_x is maximum, so is E_y
When E_x is zero, so is E_y

Trace $\mathbf{E}(t)$ in $z = 0$ plane:



$\mathbf{E}$ oscillates along line:
  state called *linear polarization*

In 3D, $\mathbf{E}$ oscillates in plane
  plane called *plane of polarization*

Snapshot of $\mathbf{E}(z, t)$ looks like cosine function
  lying in plane of polarization

Used linearly polarized light in original derivations
  only $\hat{\mathbf{x}}$ or $\hat{\mathbf{y}}$

More generally, allow any plane

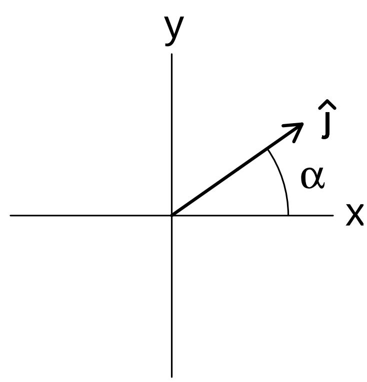
Complex notation:

$$\mathbf{E}(z,t) = E_0 \hat{\jmath} e^{i(kz-\omega t)}$$

with

$$\hat{\jmath} = \cos\alpha\,\hat{\mathbf{x}} + \sin\alpha\,\hat{\mathbf{y}}$$

for $\alpha = \tan^{-1}(E_{0y}/E_{0x})$



Plane of polarization spanned by $\mathbf{k}$ and $\hat{\jmath}$

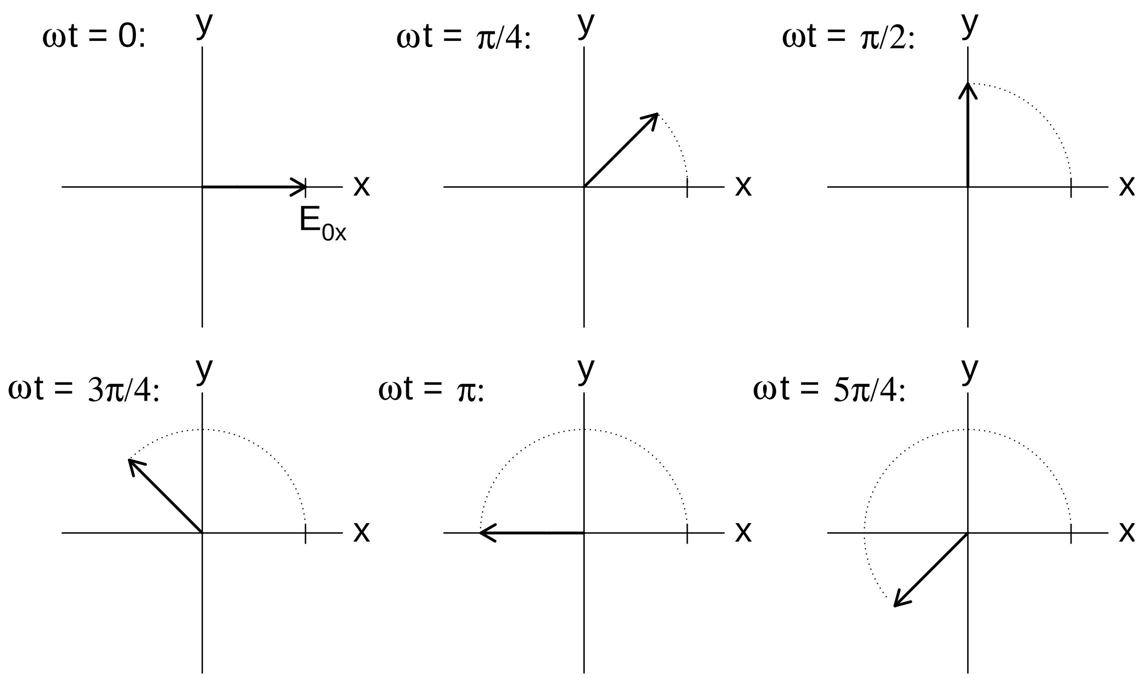
Question: How are polarization states with $\hat{\jmath}$ and $-\hat{\jmath}$ different?

Another special case: $\varepsilon = \pm\pi/2$
and $|E_{0x}| = |E_{0y}|$

Then $E_x(z,t) = |E_{0x}|\cos(kz - \omega t)$

and $E_y(z,t) = |E_{0x}|\cos(kz - \omega t \pm \pi/2)$
$= \mp|E_{0x}|\sin(kz - \omega t)$

Plot in $z = 0$ plane for $\varepsilon = +\pi/2$
So $E_x(z,t) = |E_{0x}|\cos(\omega t)$
$E_y(z,t) = |E_{0x}|\sin(\omega t)$



E rotates in circle

Called *circular polarization*

Note if $\varepsilon = -\pi/2$, $\mathbf{E}$ rotates in opposite direction

Call $\varepsilon = -\pi/2$ *right-circular* polarization (RCP)
$\varepsilon = +\pi/2$ *left-circular* polarization (LCP)

At fixed t, $\mathbf{E}(z)$ traces out helix = corkscrew
RCP: right-handed screw
LCP: left-handed screw

RCP vs. LCP very easy to mix up

LCP:

- For fixed z, $\mathbf{E}$ rotates in counter-clockwise sense
  - when light propagating toward observer

- For fixed t, $\mathbf{E}$ rotates in clockwise sense as z increases

Because of sign difference in $kz - \omega t$ factor

Also, if complex convention is $e^{i(\omega t - kz)}$

then sign of phases reversed

Fortunately, rarely need to know which is which

Complex notation:

$$\mathbf{E}(z,t) = E_0 \hat{\jmath} e^{i(kz-\omega t)}$$

with

$$\hat{\jmath} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}}) \qquad \text{(RCP)}$$

$$\hat{\jmath} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}}) \qquad \text{(LCP)}$$

and $E_0 = \sqrt{2} E_{0x}$

Question: As $\mathbf{E}$ rotates, amplitude is always E_{0x}. So why so we have $E_0 = \sqrt{2} E_{0x}$?

Linear and circular are only special polarizations

General case: *elliptical polarization*

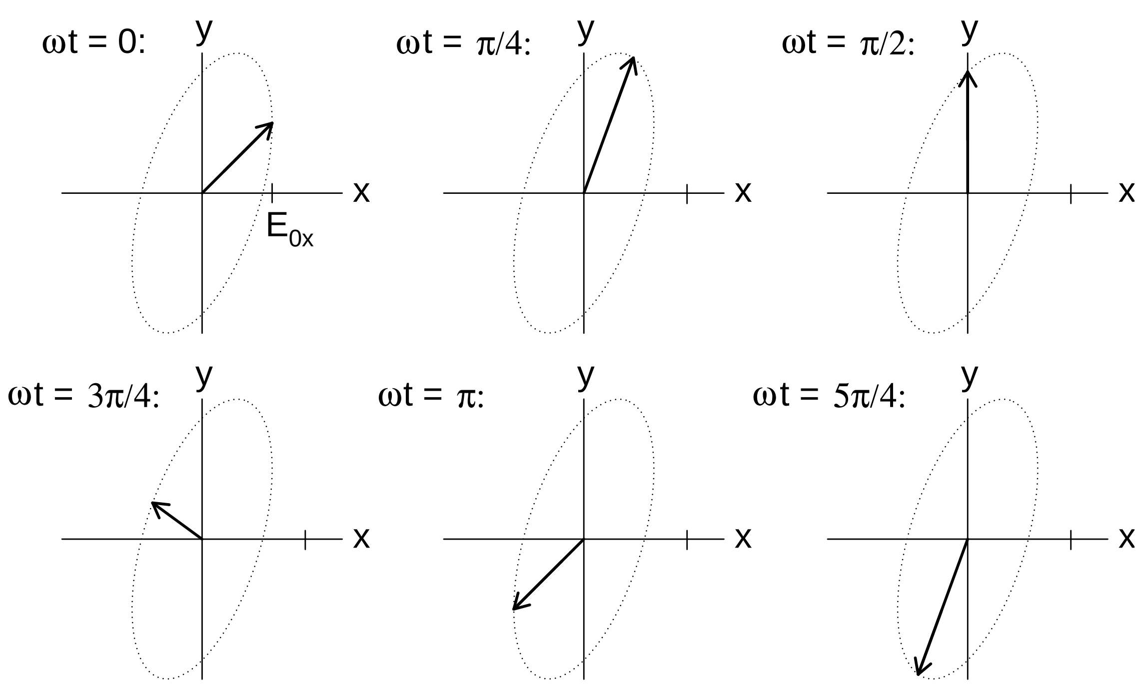
Example:

$$|E_{0y}| = 2|E_{0x}| \qquad \varepsilon = \pi/3$$

Then for $z = 0$:

$$E_x(t) = |E_{0x}| \cos(\omega t)$$
$$E_y(t) = 2|E_{0x}| \cos(\omega t - \pi/3)$$

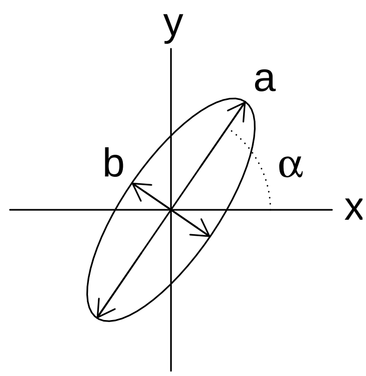
Traces out ellipse:



Equation of ellipse from $|E_{0x}|$, $|E_{0y}|$ and ε:

$$\frac{E_x^2}{|E_{0x}|^2} + \frac{E_y^2}{|E_{0y}|^2} - \frac{2E_xE_y\cos\varepsilon}{|E_{0x}||E_{0y}|} = \sin^2\varepsilon$$

Characterize by angle α and eccentricity $e = a/b$



Define $p = |E_{0y}|/|E_{0x}|$

Then angle of axes α:

$$\tan 2\alpha = \frac{2p\cos\varepsilon}{1-p^2} \qquad \text{(Note: axis ambiguous)}$$

and eccentricity e:

$$e^2 = \frac{1+p^2+\sqrt{1+2p^2\cos 2\varepsilon + p^4}}{1+p^2-\sqrt{1+2p^2\cos 2\varepsilon + p^4}}$$

For example shown, $\alpha = 73.1^\circ$ and $e = 2.48$

These formulas hard to find!

General properties:

- eccentricity = 0 for $\varepsilon = 0$
  (linear polarization)

- eccentricity max for $\varepsilon = \pm\pi/2$
  (circ. if $|E_{0x}| = |E_{0y}|$)

- right-handed rotation for $\varepsilon < 0$
- left-handed for $\varepsilon > 0$

Complex notation:

$$\mathbf{E}(z,t) = E_0 \hat{\jmath} e^{i(kz-\omega t)}$$

with

$$E_0 = \sqrt{|E_{0x}|^2 + |E_{y0}|^2}$$

$$\hat{\jmath} = \cos\beta\,\hat{\mathbf{x}} + e^{i\varepsilon}\sin\beta\,\hat{\mathbf{y}}$$

$$\tan\beta = p = \frac{|E_{0y}|}{|E_{0x}|}$$

Note β not the same as ellipse angle α

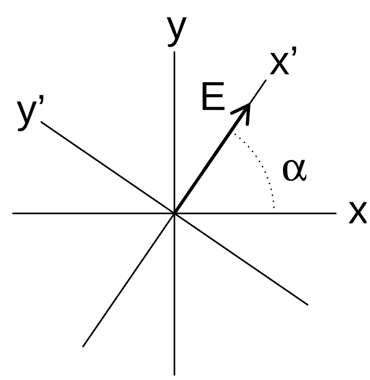
Find $\tan(2\alpha) = \tan(2\beta)\cos(\varepsilon)$

Choice of Basis

So far have used x, y coordinates

Problems often easier in different coords

Example:
linearly polarized light at angle α



Define $x' = x \cos\alpha + y \sin\alpha$

$$y' = -x \sin\alpha + y \cos\alpha$$

Then light polarized along x'

$$\mathbf{E}(z,t) = E_0 \hat{\mathbf{x}}' e^{i(kz-\omega t)}$$

Set of unit vectors = *basis*

Can also use complex $\hat{\jmath}$'s as basis

Most often use circular states

$$\hat{\mathbf{e}}_{\mathcal{R}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}})$$

$$\hat{\mathbf{e}}_{\mathcal{L}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})$$

Example: can write $\hat{\mathbf{x}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{e}}_{\mathcal{R}} + \hat{\mathbf{e}}_{\mathcal{L}})$

Useful if circ. polarizations are important for you

Another example: arb. elliptical state
= sum of linear and circular states

Say $\hat{\jmath} = \cos\beta\,\hat{\mathbf{x}} + e^{i\varepsilon}\sin\beta\,\hat{\mathbf{y}}$

Then

$$\begin{aligned}\hat{\jmath} &= \cos\beta\,\hat{\mathbf{x}} + \cos\varepsilon\sin\beta\,\hat{\mathbf{y}} + i\sin\varepsilon\sin\beta\,\hat{\mathbf{y}} \\ &= (\cos\beta - \sin\varepsilon\sin\beta)\,\hat{\mathbf{x}} + \cos\varepsilon\sin\beta\,\hat{\mathbf{y}} \\ &\qquad + (\sin\varepsilon\sin\beta\,\hat{\mathbf{x}} + i\sin\varepsilon\sin\beta\,\hat{\mathbf{y}}) \\ &= (\cos\beta - \sin\varepsilon\sin\beta)\,\hat{\mathbf{x}} + \cos\varepsilon\sin\beta\,\hat{\mathbf{y}} \\ &\qquad + \sqrt{2}\sin\varepsilon\sin\beta\,\hat{\mathbf{e}}_{\mathcal{L}}\end{aligned}$$

First line: linear at

$$\tan\alpha = \frac{j_y}{j_x} = \frac{\cos\varepsilon\sin\beta}{\cos\beta - \sin\varepsilon\sin\beta}$$

Second line: LHC

Optical elements have simple effect in some bases,
complicated in others

Useful to go back and forth

For complex bases, need orthogonality condition

Need two basis vectors $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$, with $\hat{\mathbf{e}}_1 \perp \hat{\mathbf{e}}_2$

For complex vectors, $\perp$ means $\hat{\mathbf{e}}_1^* \cdot \hat{\mathbf{e}}_2 = 0$

Example: if $\hat{\mathbf{e}}_1 = \frac{\sqrt{3}}{2}\hat{\mathbf{x}} + i\frac{1}{2}\hat{\mathbf{y}}$

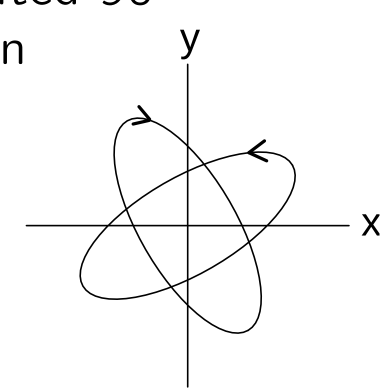
then $\hat{\mathbf{e}}_2 = \frac{1}{2}\hat{\mathbf{x}} - i\frac{\sqrt{3}}{2}\hat{\mathbf{y}}$

since $\hat{\mathbf{e}}_1^* \cdot \hat{\mathbf{e}}_2 = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

In general, $\sin\beta\hat{\mathbf{x}} - e^{i\varepsilon}\cos\beta\hat{\mathbf{y}}$ is orthogonal to $\cos\beta\hat{\mathbf{x}} + e^{i\varepsilon}\sin\beta\hat{\mathbf{y}}$

Graphically:

Orthogonal polarizations rotated 90°
and opposite sense of rotation



Question: What state is orthogonal to LHC polarization, and does it satisfy $\hat{\mathbf{e}}^* \cdot \hat{\mathbf{e}}_{\mathcal{L}} = 0$?

Partially Polarized Light (Hecht 8.1.4)

Previously introduced idea of coherence

Two waves $|A_1|e^{i(kz-\omega t+\phi_1)}$ and $|A_2|e^{i(kz-\omega t+\phi_2)}$
are *coherent* if phase diff $\phi_1 - \phi_2$ is constant

Constant = constant over time scale of interest

Say single wave is coherent if ϕ_1 is constant

Most natural light sources are incoherent

Coherence affects polarization

For totally incoherent light,

$$E_x(z,t) = |E_{0x}| \cos(kz - \omega t + \phi_x)$$
$$E_y(z,t) = |E_{0y}| \cos(kz - \omega t + \phi_y)$$

ϕ_x and ϕ_y vary randomly

All polarization effects average out:
Say light is *unpolarized*

Light can be incoherent but polarized

Suppose $E_x(z,t) = |E_{0x}| \cos(kz - \omega t + \phi)$
$E_y(z,t) = |E_{0y}| \cos(kz - \omega t + \phi + \varepsilon)$

with ϕ fluctuating but ε constant

Then E_x and E_y components fluctuate together

- Alternatively, could just have $E_{0y} = 0$

Either way, see polarization effects

Unpolarized light =
"mixture" of any two orthogonal states

Add irradiances of each, not fields

$I_{\text{tot}} = I_1 + I_2$

If system transmits $\hat{\mathbf{e}}_1$ with transmittance T_1,
$\hat{\mathbf{e}}_2$ with transmittance T_2

Get $I_{\text{out}} = T_1 I_1 + T_2 I_2$

No interference effects

Example:
sunlight = 50% linear $\|$ + 50% linear $\perp$

Transmission through surface $\langle T \rangle = \frac{1}{2}(T_\| + T_\perp)$

Doesn't matter what is $\hat{\mathbf{x}}$, what is $\hat{\mathbf{y}}$

Or: sunlight = 50% RHC + 50% LHC

Suppose some material absorbs all RHC:
    Get 50% transmittance

As before, work in whatever basis is easiest
- Here, don't need to recalculate state

Connection to Quantum Mechanics

Mathematics of polarization
    = math of quantum two-level system

Examples:
- Electron in magnetic field $\Leftarrow$
- Two atomic levels coupled by field
- Single proton in NMR

Doesn't mean that light is quantum mechanical!
- means that two-level systems are classical

Apply QM understanding to light

- $\mathbf{k} \leftrightarrow \mathbf{B}$ (magnetic field)
- $\hat{\jmath} \leftrightarrow |\psi\rangle$
- basis states $\leftrightarrow$ basis states
- LHC $\rightarrow$ spin up along z
- RHC $\rightarrow$ spin down along z
- Linear polarized along x = spin along x
- Unpolarized light = mixture states
  (w/ density matrix)

Applies to optical devices
= measurement or unitary operators

Connect to photon optics at end of course

Summary:

- Linear polarization: $\mathbf{E}$ oscillates in plane
  $\hat{\jmath} = \cos\alpha\,\hat{\mathbf{x}} + \sin\alpha\,\hat{\mathbf{y}}$
- Circular polarization: $\mathbf{E}$ winds in helix
  $\hat{\jmath} = (\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})/\sqrt{2}$
- More generally, $\mathbf{E}$ traces out ellipse
  $\hat{\jmath} = \cos\beta\,\hat{\mathbf{x}} + e^{i\varepsilon}\sin\beta\,\hat{\mathbf{y}}$
- Work in whatever basis is convenient
  - Just like QM
- Unpolarized light:
  mixture of orthogonal states