Phys 531 Lecture 20 11 November 2004

Polarizers and Retarders

Last time, discussed basics of polarization

Linear, circular, elliptical states

Describe by polarization vector $\hat{\jmath}$

Today:
Describe elements used to manipulate polarization

Outline:

- Polarizers
  - Reflection, scattering, dichroism
  - Calculations with polarizers
- Birefringence
- Retarders

Next time: Jones calculus
- powerful tool for doing calculations
- will be helpful for hw

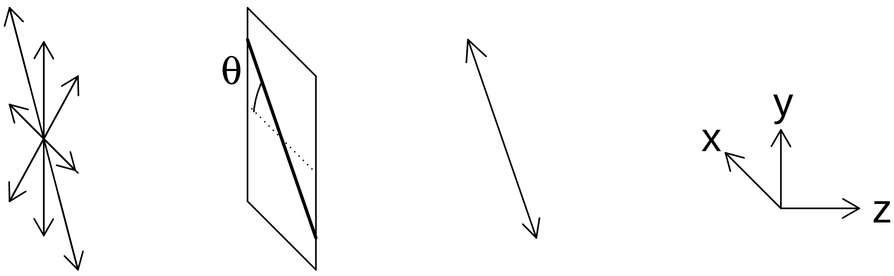
Polarizers (Hecht 8.2)

Most natural light sources are unpolarized

Obtain polarized light with *polarizer*
= "filter" passing only one polarization state

Usually transmit linear polarization

Plane of polarization given by *transmission axis*



Ideal polarizer:
Transmission for $\hat{\jmath} \parallel$ axis = 1
Transmission for $\hat{\jmath} \perp$ axis = 0

If axis at angle θ, write
$\mathbf{a} = \cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}}$

If incident light has polarization $\hat{\jmath}$:
transmit component of $\hat{\jmath} \parallel \mathbf{a}$: $\hat{\jmath}^* \cdot \mathbf{a}$

So $\boxed{T = |\hat{\jmath}^* \cdot \mathbf{a}|^2}$

If $\hat{\jmath}$ linearly polarized $\hat{\jmath} = \cos\alpha\,\hat{\mathbf{x}} + \sin\alpha\,\hat{\mathbf{y}}$

Then $\hat{\jmath}^* \cdot \mathbf{a} = \hat{\jmath} \cdot \mathbf{a} = \cos\theta\cos\alpha + \sin\theta\sin\alpha$
$= \cos(\theta - \alpha)$

Gives *Malus's Law*:

For linear polarization incident on polarizer,
$I_{\text{out}} = I_{\text{in}} \cos^2(\theta - \alpha)$

$\theta - \alpha$ = angle difference between transmission axis and incident plane of polarization

But $T = |\hat{\jmath}^* \cdot \mathbf{a}|^2$ is more general
works for any incident polarization state

Question: Could $\mathbf{a}$ be complex? What would it mean if it were?

Example:

If $\hat{\jmath}_{\text{inc}} = \hat{\mathbf{e}}_{\mathcal{R}} = \dfrac{\hat{\mathbf{x}} - i\hat{\mathbf{y}}}{\sqrt{2}}$, what is transmission
through linear polarizer at angle θ?

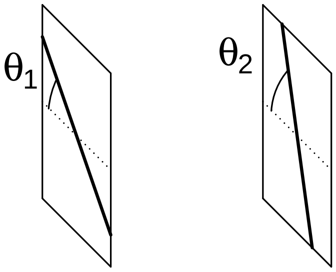
Have $\hat{\jmath}^* \cdot \mathbf{a} = \dfrac{\cos\theta + i\sin\theta}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} e^{i\theta}$

So $T = \dfrac{1}{2}|e^{i\theta}|^2 = \dfrac{1}{2}$ independent of θ

Makes sense, circ polarization symmetric in θ

Note, light exitting ideal polarizer has $\hat{\jmath}_{\text{out}} = \mathbf{a}$

If two polarizers: first at θ_1, second at θ_2



Output of first = polarized along θ_1

Transmission of second = $\cos^2(\theta_2 - \theta_1)$

Real polarizers aren't perfect:

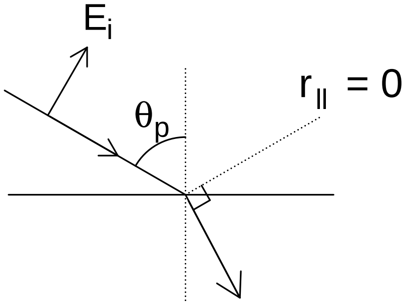
- Transmission for $\hat{\jmath} \parallel \mathbf{a} = T_0 < 1$
  (loss)
- Transmission for $\hat{\jmath} \perp \mathbf{a} = \epsilon > 0$
  (leakage)
- Output light not exactly polarized along $\mathbf{a}$
  (rarely specified)

Values depend on type of polarizer

Discuss types of polarizers

Constructing Polarizers (Hecht 8.3–8.6)

Already know one way to polarize light:
use Brewster's angle



When TM polarized light incident at angle
$\theta_p = \tan^{-1}(n_t/n_i)$

Get $r_\parallel = 0$

Two ways to make polarizer:

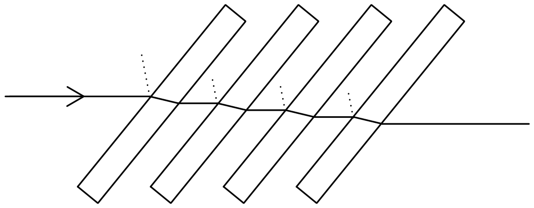
- Use reflected light: get $\perp$ component

Then loss is very high:
- glass, get $R_\perp \approx 0.2 \rightarrow$ lose 80%

Also, leakage is fairly high:
- hard to control angle accurately

- Better: use transmitted light
  and many surfaces

Pile of plates polarizer:



Each surface transmits all of $I_\parallel$
 and fraction $T_\perp$ of $I_\perp$
 for glass, $T_\perp \approx 0.8$

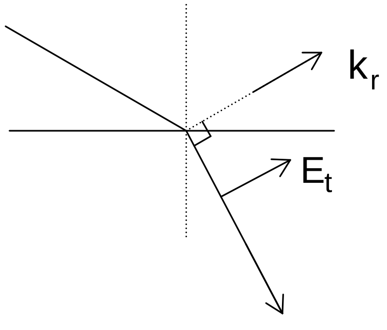
For N plates, total $\perp$ transmission $= T_{\perp\text{tot}} = T_\perp^{2N}$

Say glass plates, $N = 10$: $T_{\perp\text{tot}} = 0.01$

Typically get total $T_{\parallel\text{tot}} = 0.5$

Pile of plates simple and robust
But often awkward to use:
 thick, requires collimated light

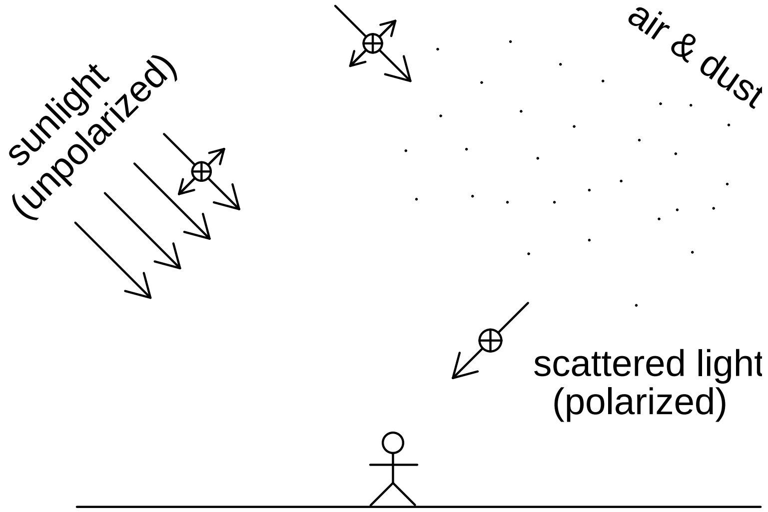
Based on scattering properties:
 Recall Brewster angle when $\mathbf{k}_{\text{ref}} \parallel \mathbf{E}_{\text{trans}}$



Atoms in glass can't radiate $\parallel \mathbf{E}$
 (Charges radiate $\perp$ acceleration)

Scattered light is generally polarized

Example: light from sky



Not typically useful as polarizer

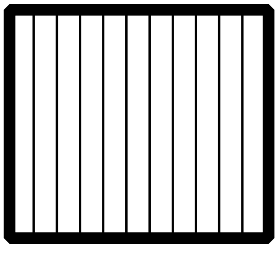
Dichroism (Hecht 8.3)

Dichroism = selective absorption of one (linear) polarization

Clearly useful for polarizers

Example: microwave polarizer

Array of parallel wires
    spacing $\ll \lambda$



$\mathbf{E}$ aligned with wires: drive current
    resistance → power dissipation
        → absorption

$\mathbf{E} \perp$ wires: little current, no absorption

Acts as a polarizer: transmits only $\mathbf{E} \perp$ wires

Watch out: graphically, want to picture vertical $\mathbf{E}$ "squeezing" through slots

Actual effect is just the opposite!

Optical version: wires → long polymer chains
    Embed in clear plastic
    Stretch plastic to align chains

Material called *polaroid*

Most common polarizer

Great for demos!

Characteristics of polaroid:

- Somewhat lossy: $T_0 \approx 0.7$
- Low leakage $\epsilon \approx 10^{-3}$
- Work best for visible light
- Cheap: $1 for 5 cm square

Important restriction: limited to low power
(plastic can melt)

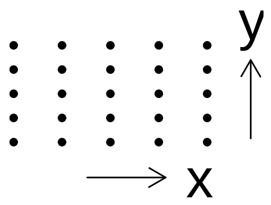
Don't use with higher intensity laser beams
max $I \approx 1$ W/cm^2

Birefringence (Hecht 8.4)

Best polarizers based on *birefingence*
- Property of certain crystals

Generally, different directions not equivalent

Possible crystal lattice:

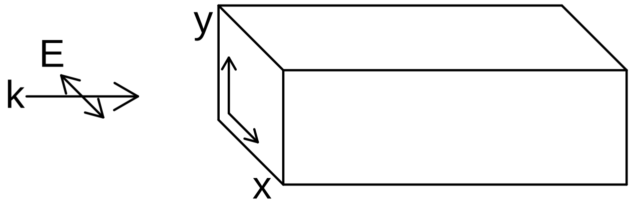


x and y axes different

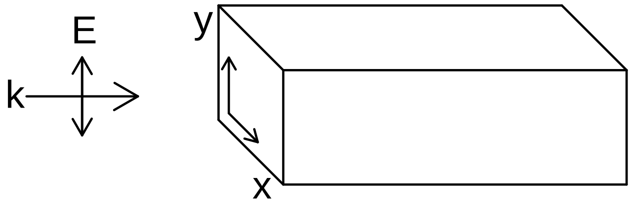
Note x and y = symmetry axes of crystal

In asymmetric crystal, index of refraction n
depends on direction of $\mathbf{E}$

If $\mathbf{E}$ along x then have $n = n_x$:



If $\mathbf{E}$ along y then $n = n_y$:



All crystals have three basic symmetry axes
for now, label x, y and z

Call n_x, n_y, n_z = *principle indices of refraction*

Three different kinds of crystals:

- isotropic: $n_x = n_y = n_z$
  - not birefringent
- uniaxial: $n_x = n_y \neq n_z$
  - z axis special: called optic axis
- biaxial: $n_x \neq n_y \neq n_z$
  - optical properties complicated

Question: Can a liquid be birefringent?

Focus on uniaxial:
Symmetry like a cylinder: x and y interchangable

Terminology: call $n_x, n_y = n_o$
ordinary index

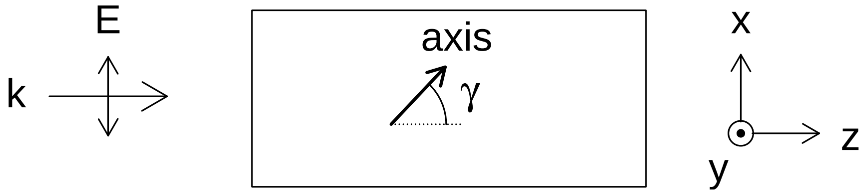
Call $n_z = n_e$: extraordinary index

Common optical materials:
- Calcite: $n_o = 1.658$, $n_e = 1.486$
- Quartz: $n_o = 1.544$, $n_e = 1.553$

Other examples: ice, mica, sapphire, $\mathrm{LiNbO_3}$

What happens if $\mathbf{k}$ is not along a crystal axis?

Example:



Light propagates along z
- $\mathbf{E}$ along x
- optic axis at angle γ in xz-plane

Question: What is index if $\mathbf{E}$ along y?

For $\mathbf{E}$ along x, get effective index n_{eff}:

$$\boxed{\frac{1}{n_{\text{eff}}^2} = \frac{\cos^2\gamma}{n_o^2} + \frac{\sin^2\gamma}{n_e^2}}$$

If $\gamma = 0$, $n_{\text{eff}} = n_o$
if $\gamma = 90^\circ$, $n_{\text{eff}} = n_e$

Otherwise n_{eff} between n_o and n_e

Derivation a bit hard, won't go through
See Klein and Furtak §9.4

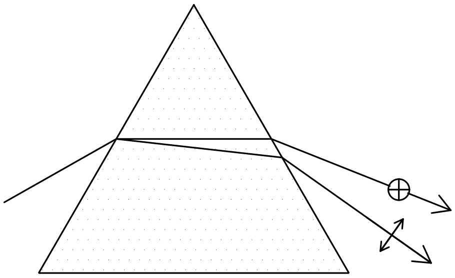
Probably cover in Phys 532

Upshot:

In birefringent materials, n depends on polarization

Simple polarizer:

Calcite prism, axis ⊥ to page



⊥ and ∥ polarizations have different n's
Deflected by different amounts

Separate outputs with lens or free propagation

Example of a *polarizing beam splitter*

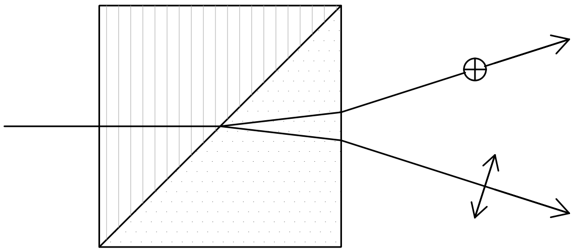
= polarizer with two outputs
one for each state

But not a good design:

- Deflection depends on λ
- Significant reflection from surfaces
- Large common deflection inconvenient

Improve by putting two prisms together

Wollaston prism:



Typical angular separation = 15-20°

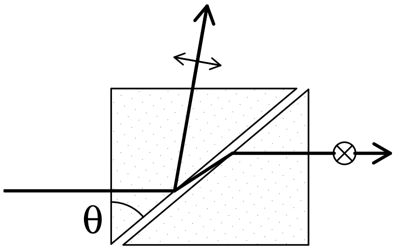
Good performance:

- Loss $\approx$ 10%, or 1% if AR coated
- Leakage $\sim 10^{-5}$
- Works at high power

Several other designs, see optics catalogs

Another good design: Glan-Thompson

Uses total internal reflection



Again calcite, with optical axis ⊥ page

Choose prism angle so that $n_e \sin\theta < 1 < n_o \sin\theta$
$\theta = 40^\circ$ works

Then o-light is TIR, e-light is transmitted

(Gap is too big for frustrated TIR)

Performance similar to Wollaston
- low loss, low leakage
- high power capacity

Advantage: larger beam separation
no deviation of e beam

Wollaston and Glan-Thompson expensive
\$300-\$500 or more

Retarders (Hecht 8.7)

Use polarizers to make linear polarized light

What about circular or elliptical?

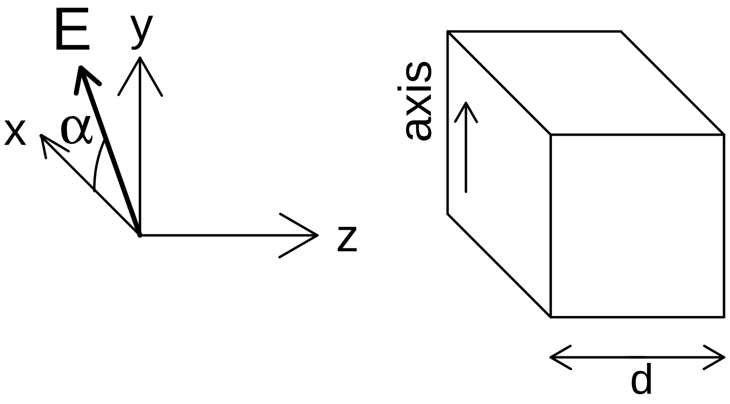
Use *retarder*

Most common retarder = wave plate

Again based on birefringence

Suppose linear polarized light, angle α

Incident on uniaxial crystal with axis vertical
    Crystal thickness d



Coordinates as shown: $z = 0$ at front of crystal

Incident wave has $\hat{\jmath} = \cos\alpha\,\hat{\mathbf{x}} + \sin\alpha\,\hat{\mathbf{y}}$

$$\mathbf{E}_{\text{inc}} = E_0 \hat{\jmath} e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

In crystal $k \to nk_0$
different for E_x and E_y components

So

$$\mathbf{E}_{\text{crystal}} = E_0\left(\cos\alpha\, e^{in_o k_0 z}\,\hat{\mathbf{x}} + \sin\alpha\, e^{in_e k_0 z}\,\hat{\mathbf{y}}\right) e^{-i\omega t}$$

At output $z = d$:

$$\begin{aligned}\mathbf{E}_d &= E_0\left(\cos\alpha\, e^{in_o k_0 d}\,\hat{\mathbf{x}} + \sin\alpha\, e^{in_e k_0 d}\,\hat{\mathbf{y}}\right) e^{-i\omega t}\\ &= E_0 e^{in_o k_0 d}\left[\cos\alpha\,\hat{\mathbf{x}} + \sin\alpha\, e^{i(n_e-n_o)k_0 d}\,\hat{\mathbf{y}}\right] e^{-i\omega t}\end{aligned}$$

Define $\hat{\jmath}_{\text{out}} = \cos\alpha\,\hat{\mathbf{x}} + \sin\alpha\, e^{i(n_e-n_o)k_0 d}\,\hat{\mathbf{y}}$

Then after crystal, have

$$\begin{aligned}E(z,t) &= E_0 e^{in_o k_0 d}\,\hat{\jmath}_{\text{out}}\, e^{i[k(z-d)-\omega t]}\\ &= E_0 e^{i(n_o-1)k_0 d}\,\hat{\jmath}_{\text{out}}\, e^{i(kz-\omega t)}\\ &= E_0'\,\hat{\jmath}_{\text{out}}\, e^{i(kz-\omega t)}\end{aligned}$$

Get plane wave out with

$$\hat{\jmath} = \cos\alpha\,\hat{\mathbf{x}} + \sin\alpha\, e^{i\varepsilon}\,\hat{\mathbf{y}}$$

with $\boxed{\varepsilon = (n_e - n_o)k_0 d} \equiv$ *retardance*

Set d to achieve desired ε value

By adjusting α and ε,
make arbitrary polarization state

Example: $kd(n_e - n_o) = \pi/2$
Then $d(n_e - n_o) = \lambda/4$: call *quarter-wave plate*

Get $\hat{\jmath}_{\text{out}} = \cos\alpha\,\hat{\mathbf{x}} + e^{i\pi/2}\sin\alpha\,\hat{\mathbf{y}}$

$$= \cos\alpha\,\hat{\mathbf{x}} + i\sin\alpha\,\hat{\mathbf{y}}$$

For $\alpha = \pm 45^\circ$, $\hat{\jmath}_{\text{out}} = \dfrac{\hat{\mathbf{x}} \pm i\hat{\mathbf{y}}}{\sqrt{2}}$

Make LHC and RHC polarizations

Main use of quarter-wave plate:
convert linear to circular polarization

Other common configuration: $\varepsilon = \pi$

Then $d(n_e - n_o) = \lambda/2$: *half-wave plate*

Have $e^{i\pi} = -1$ so
$\hat{\jmath}_{\text{out}} = \cos\alpha\,\hat{\mathbf{x}} - \sin\alpha\,\hat{\mathbf{y}}$

Changes polarization angle $\alpha \to -\alpha$

Suppose $\alpha_{\text{in}} = 45^\circ$

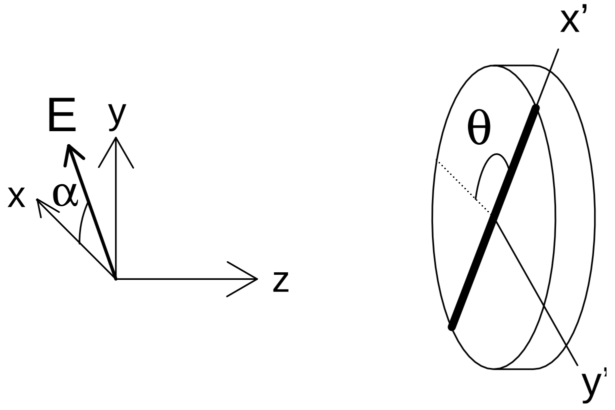
Then $\alpha_{\text{out}} = -45^\circ$:
Rotated by 90°: orthogonal to input

Use half-wave plate by rotating it

how to calculate effect?

$\Rightarrow$ need to express input in crystal basis

Define x' = axis of waveplate



Then

$$\hat{\mathbf{x}}' = \cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}}$$
$$\hat{\mathbf{y}}' = \cos\theta\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{x}}$$

and $\hat{\jmath} = \cos\alpha\,\hat{\mathbf{x}} + \sin\alpha\,\hat{\mathbf{y}}$

Calculate $j_{x'} = \hat{\jmath} \cdot \hat{\mathbf{x}}'$

$$= \cos\alpha\cos\theta - \sin\alpha\sin\theta = \cos(\alpha - \theta)$$

and $j_{y'} = \hat{\jmath} \cdot \hat{\mathbf{y}}'$

$$= -\cos\alpha\sin\theta + \sin\alpha\cos\theta = \sin(\alpha - \theta)$$

Or $\hat{\jmath} = \cos\alpha'\,\hat{\mathbf{x}}' + \sin\alpha'\,\hat{\mathbf{y}}'$ for $\alpha' = \alpha - \theta$

Effect of half wave plate is $\alpha' \to -\alpha'$

So $\alpha_{\text{out}} - \theta = -(\alpha_{\text{in}} - \theta)$
$\alpha_{\text{out}} = 2\theta - \alpha_{\text{in}}$

As θ adjusted, output polarization rotates by 2θ

Main use of half-wave plate:
rotate linear polarization by arbitrary angle

Question: What happens when $\theta = \alpha_{\text{in}}$? What is physically happening in this situation?

One problem: waveplates very thin

Quarter wave plate: $d = \lambda/4(n_e - n_o) \sim \lambda$

Often make $\varepsilon = \pi/4 + 2\pi m$ for integer m:
get same effect

Called *multiple order waveplate*
typical $m \approx 10$

Most common waveplate material: quartz
cost $200 for 1-cm diameter plate

Cheaper: plastic ($5 for 5 cm square)
- ε less accurate
- distorts laser beams

Terminology:

Fast axis: axis of wave plate with lower n
Slow axis: axis of wave plate with higher n

Doesn't really matter which is optical axis
depends on whether $n_o > n_e$ ("negative" crystal)
or $n_o > n_e$ ("positive" crystal)

Other ways to make retarders (Hecht pg 356-357):

- Fresnel rhomb: use phase shift from TIR
- Babinet-Soliel compensator:
  waveplate with variable thickness

Summary:

- Polarizers transmit one polarization
  $T = T_0|\hat{\jmath}^* \cdot \mathbf{a}|^2$
- Most polarizers dichroic or birefringent
  Birefringent better, more $
- Birefringence: n depends on $\hat{\jmath}$
  - Uniaxial crystal: one special direction
- Retarders use birefringence
  - Quarter-wave plate: make circ polarization
  - Half-wave plate: rotate linear polarization