Phys 531 Lecture 26 7 December 2004

Quantum Optics

Last time, finished coherence theory

Saw that extended sources introduce incoherence
even when $E(t)$ is regular

Interference only easy to see with
monochromatic point source ≈ laser

Today: introduce quantum optics
Whole new way to look at light

Outline:

- Photon optics
- Quantum field theory
- Quantum states of light

Photon optics pretty simple

Field theory pretty hard
Really need quantum to understand
Not included on final exam

Want more?
Take Phys 888 Quantum Optics next semester!

Next time: review . . . bring questions!

Photon Optics (Hecht 3.3.3)

Simple version of quantum theory

Say that light is really composed of particles
particles called *photons*

Energy of photon = $\hbar\omega$
$\hbar$ = Planck's constant = 1.054×10^{-34} J s
ω = oscillation frequency of light
(Don't worry about what is oscillating for now)

Polychromatic light:
many photons with different ω's

- Pulse of light with energy U (in J)

  contains $N = \dfrac{U}{\hbar\omega}$ photons

N = *photon number*

- Beam with power P (in W = J/s)

  delivers $\Phi = \dfrac{P}{\hbar\omega}$ photons/s

Φ = *photon flux*

- Beam with irradiance $I(\mathbf{r})$ (in W/m^2)

  has $\phi = \dfrac{I}{\hbar\omega} \; \dfrac{\text{photons/s}}{\text{m}^2}$ at $\mathbf{r}$

ϕ = *photon flux density*

Also light with energy density $u(\mathbf{r})$ (in J/m^3)

has photon density $\frac{u}{\hbar\omega}$ $\left(\frac{\text{photons}}{\text{m}^3}\right)$

Example:

Sunlight has an irradiance of about 250 W/m^2

Assume average wavelength = 500 nm

Then average $\hbar\omega = 4 \times 10^{-19}$ J

Photon flux density = 6×10^{20} photons/(s m^2)

Looking at sun, pupil area $\approx 10^{-6}$ m^{-2}

collect about 6×10^{14} photons in 1 s

Photons are not "classical" particles!

Propagate according to wave equation

not Newton's laws

Procedure: use wave techniques to calculate $I(\mathbf{r})$

Then I gives flux density

Imagine photons "follow" wave

like surfers in ocean

But photons and wave are inseparable

Best to interpret probabilistically:

Average number of photons in volume $d^3r =$

$$\langle N \rangle = \frac{u(\mathbf{r})\, d^3r}{\hbar\omega} = \frac{I(\mathbf{r})\, d^3r}{\hbar\omega c}$$

If $\langle N \rangle \ll 1$, interpret as probability to
find one photon

Important:

No definite trajectory for individual photons

Picture surfers scrambling back and forth on wave

Avoids "two-slit" paradox:

In two slit interferometer, which slit does photon pass through?

Correct interpretation:
It doesn't matter

Wave passes through both

Interference pattern affects where photon ends up

Question: What if wave passes through beam spitter, and outputs are separated by a large distance? Does it make sense to ask where photon is?

If wave determines photon distribution,
why use photons?

Because detectors see photons, not waves

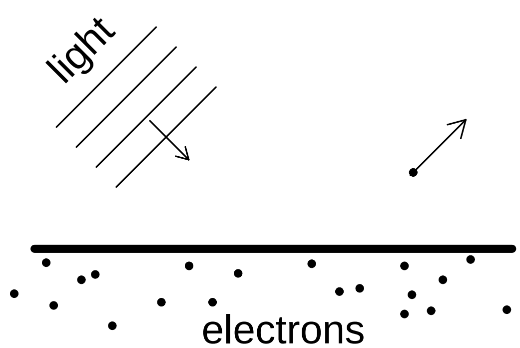
Or: light can only transfer energy in units of $\hbar\omega$

So photons important whenever light is emitted or absorbed

Example: photoelectric effect

Shine light on metal

Electrons absorb energy from light and escape

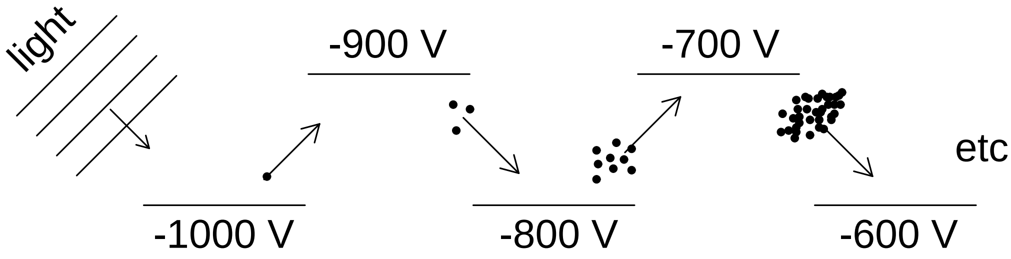


Find that maximum electron energy $= \hbar\omega$
$=$ max energy absorbed from photon

Doesn't depend on irradiance, just ω

Another example: photomultiplier tube

Light hits metal plate, detaches single electron



Accelerate electron to second plate:
detach more electrons

Cascade many plates, get big current pulse

See blip on output: detection of one photon

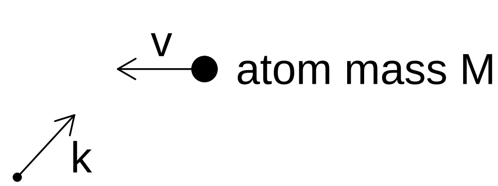
Other photon properties:

• Momentum $\mathbf{p} = \hbar\mathbf{k}$ (Hecht 3.3.4)

Interesting effect: suppose atom absorbs photon
makes internal transition $U_0 \to U_1$

Say atom velocity before absorption $= \mathbf{v}$

Velocity after absorption $= \mathbf{v} + \hbar\mathbf{k}/M$
gets kick from photon



Initial energy: $U_{\text{tot}} = U_0 + \frac{1}{2}Mv^2 + \hbar\omega$

Final energy: $U_{\text{tot}} = U_1 + \frac{1}{2}M\left(\mathbf{v} + \frac{\hbar\mathbf{k}}{M}\right)^2$

Energy conserved, so

$$\omega = \frac{U_1 - U_0}{\hbar} + \frac{\hbar k^2}{2M} + \mathbf{v}\cdot\mathbf{k}$$

First term: standard QM

Second term: "radiative correction"

Third term: Doppler shift

Derive Doppler effect from QM!

- Angular momentum (Hecht 8.1.5)

Circular polarization states have definite spin

Right circular: $\mathbf{L}_{\text{photon}} = -\hbar\hat{\mathbf{k}}$

Left circular: $\mathbf{L}_{\text{photon}} = +\hbar\hat{\mathbf{k}}$

Linear polarized states
= superposition of $\hat{\mathbf{e}}_{\mathcal{R}}$ and $\hat{\mathbf{e}}_{\mathcal{L}}$

Photon equally likelty to be "spin-up" and "spin-down"

Important for transition selection rules

Quantum Field Theory

Want proper quantum theory for photon

Expect $E(\mathbf{r}) \sim$ wave function $\psi(\mathbf{r})$

Since:

- Probability $\propto |E|^2$
- E exhibits interference like ψ
- Wave equation $\sim$ Schrodinger equation
- Natural to have E complex

Intuition is right, but one problem:
number of photons is indefinite

"Normal" QM assumes N fixed
N = number of particles

Problem not just that N is unknown:

Can have quantum system in superposition

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(\,|\text{atom + photon}\rangle + |\text{excited atom}\rangle\,\Big)$$

Photon in superposition of existing or not!

Can't just modify Schrodinger eqn for N variable

Can't make quantum theory for photon
(as a particle)

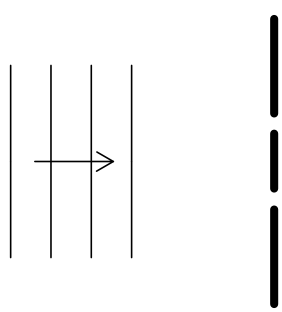
Instead, make quantum theory for field $E(\mathbf{r})$
$\Rightarrow$ quantum field theory

Idea: make $E =$ quantum operator
like $\mathbf{r}$ for electron

Say $\Psi(E) =$ state of EM field
like $\psi(\mathbf{r}) =$ state of electron

Of course, really have $\Psi[E(\mathbf{r})]$:
E is a function, not a variable

Example: two slit interferometer



From normal optics, expect solution $E(\mathbf{r})$
Wave travels through both slits, get interference

Write this solution $E_A(\mathbf{r})$

A possible quantum state is $\Psi_A(E)$:
Ψ strongly peaked at $E(\mathbf{r}) = E_A(\mathbf{r})$
Near zero for other possible $E(\mathbf{r})$'s

Could have wave that travels only through slit 1
= solution for problem with slit 2 covered

Write wave solution as $E_1(\mathbf{r})$

Similarly, $E_2(\mathbf{r})$ = solution with slit 1 covered

Imagine aperture in quantum superposition of
slit 1 covered and slit 2 covered

(Like Schrodinger's cat)

Not practical but OK for thought experiment

Then quantum state of light would be

$$\Psi_B(E) = \frac{1}{\sqrt{2}}\Big[\Psi_1(E) + \Psi_2(E)\Big]$$

where

$\Psi_1(E)$ has $E(\mathbf{r}) \approx E_1(\mathbf{r})$
$\Psi_2(E)$ has $E(\mathbf{r}) \approx E_2(\mathbf{r})$

Compare Ψ_A and Ψ_B

In Ψ_A, E is in one well-defined state
= state where wave travels through both slits

In Ψ_B, E is not in a definite state:
Entire wave travels through slit 1 or through slit 2

If we look at light on distant screen:

Ψ_A would give normal interference pattern

Ψ_B would not

Say that

E_A is *classical* superposition of E_1 and E_2

Ψ_B is *quantum* superposition of Ψ_1 and Ψ_2

Mode Expansion

To calculate in field theory, work with *modes*

Usually, mode = plane wave

Can write any $E(\mathbf{r}, t)$ as

$$E(\mathbf{r}, t) = \frac{1}{(2\pi)^3} \int A_{\mathbf{k}} e^{i(\mathbf{k}\cdot\mathbf{r} - \omega_{\mathbf{k}} t)} \, d^3k$$

so state of field uniquely specified by $\{A_{\mathbf{k}}\}$'s

Each mode $\mathbf{k}$ = independent degree of freedom

Do QM on each independently

Treat each $A_{\mathbf{k}}$ as independent quantum variable

Now a simple variable like X or P of particle

But still one issue: $A_{\mathbf{k}}$ is complex

Write $A_{\mathbf{k}} = q_{\mathbf{k}} + ip_{\mathbf{k}}$

Really *two* quantum variables $q_{\mathbf{k}}$ and $p_{\mathbf{k}}$

Turns out they are conjugate variables
just like X and P for particle

So have ordinary wave function $\psi(q_{\mathbf{k}})$ just like $\psi(X)$

Total wave function of field is

$$\Psi(E) = \prod_{\mathbf{k}} \psi_{\mathbf{k}}(q_{\mathbf{k}})$$

or as superposition of such products

$\psi(q_{\mathbf{k}})$ satisfies ordinary Schrodinger equation

Get hamiltonian from wave equation

Result: equation for quantum harmonic oscillator
frequency $\omega_{\mathbf{k}}$

So $q_{\mathbf{k}}$ and $p_{\mathbf{k}}$ = analogs of position, momentum
of particle in harmonic oscillator potential

In particular, allowed energies $= \hbar\omega_{\mathbf{k}}(N + 1/2)$
= integer number of photons!
(+ zero point energy)

Consider single mode $\mathbf{k}$ with $\mathbf{k} = k\hat{\mathbf{z}}$

Just write q, p

What do q and p correspond to physically?

Have $A = q + ip$ = amplitude of mode

Real field is Re $Ae^{i(kz-\omega t)}$

$= q\cos(kz - \omega t) - p\sin(kz - \omega t)$

So q is amplitude of wave $\sim$ cos

p is amplitude of wave $\sim$ sin

Call components "quadratures" of wave

Subtlety in analogy with harmonic oscillator:

In oscillator $x(t) = x_0 \cos\omega t$

and $p(t) = -p_0 \sin\omega t$

So q and p really analogous to x_0, p_0

- oscillation already factored out

More convenient:

q and p don't explicitly depend on $\mathbf{r}$, t

Note for experts:

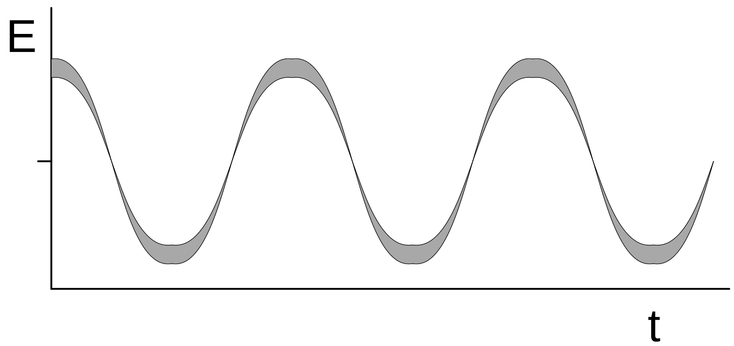
Means we're working in Heisenberg picture

Quantum States of Light

But q and p are quantum operators:
Don't have definite values

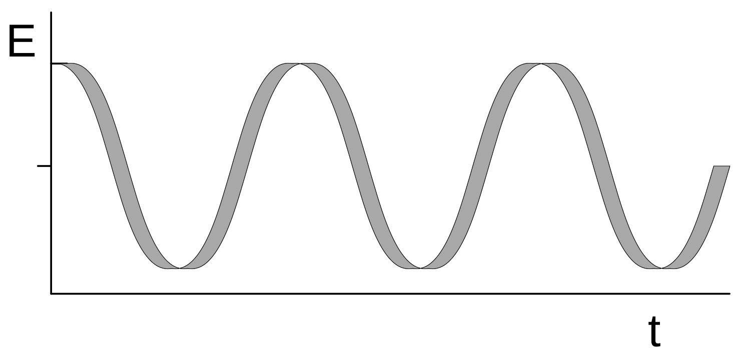
Uncertainties satisfy $\Delta q \Delta p \geq$ const

Can draw picture:
Say $\langle q \rangle$ large, $\langle p \rangle = 0$ and $\Delta q \gg \Delta p$



Amplitude uncertain

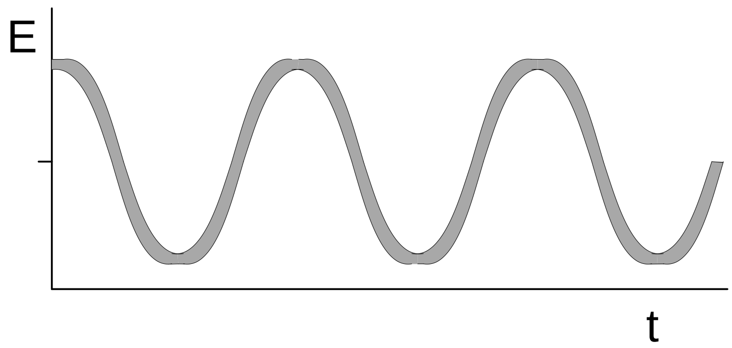
Or, if $\Delta q \ll \Delta p$:



Phase uncertain

Question: What if $\langle p \rangle$ and Δq were large, and $\langle q \rangle$ and Δp were small? Would the phase or amplitude be more certain?

Normally, have $\Delta q = \Delta p$



Uncertainty in both amplitude and phase

This is typical state produced by laser:
called "coherent state"

Fluctuations readily measured with photodiode

Label coherent state by $\alpha = \langle q \rangle + i \langle p \rangle$
$=$ expectation value of field amplitude

Expectation value of energy $\propto |\alpha|^2$

Photon number $\langle N \rangle \propto |\alpha|^2/\hbar\omega$

Get uncertainty in N: $\dfrac{\Delta N}{\langle N \rangle} = \dfrac{1}{\sqrt{\langle N \rangle}}$

Also in phase $\Delta \phi = \dfrac{1}{\sqrt{\langle N \rangle}}$

Simple explanation:
For low $\langle N \rangle$, don't collect many photons:
field hard to measure

If $\Delta q \neq \Delta p$, called "squeezed state"

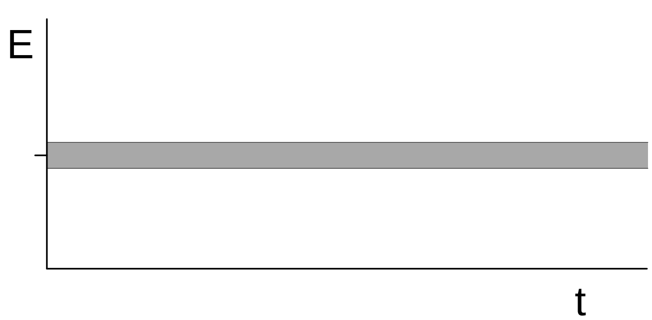
Useful for precision interferometry:
- reduce noise in variable measured
- increase noise in variable not measured

Generate squeezed states in nonlinear crystals
Rather difficult to achieve

Can have coherent state with $\alpha = 0$

Then $\langle E \rangle = 0$: no photons present
Called *vacuum state*

But still have fluctuations $\Delta q = \Delta p$:

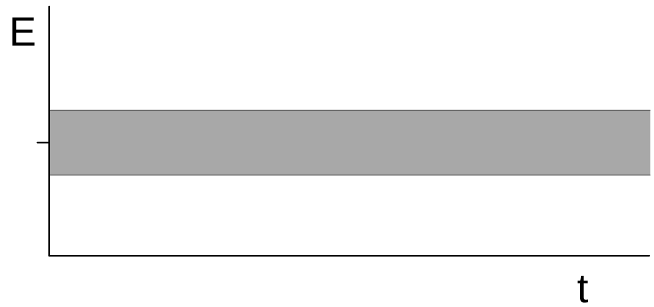


Even these fluctuations can be measured:
called vacuum noise
Helps explain spontaneous emission

Another possible state: Fock state $|N\rangle$

Has definite energy $(N + 1/2)\hbar\omega$
$\rightarrow$ mode contains N photons

Generally both Δq and Δp large $\propto \sqrt{N}$



Note vacuum state is also a Fock state
- hard to make any others

Entanglement

Complete EM field consists of many modes

Usually, most have few or no photons
= in vacuum state: ignore

Suppose only two modes occupied $\mathbf{k}_1$ and $\mathbf{k}_2$

Simple quantum state:

$$|\Psi\rangle = |1\rangle_1 \, |1\rangle_2 \times |\text{vacuum}\rangle_{\text{rest}}$$

Describes state with one photon in mode 1
one photon in mode 2

Hard to make, easy to understand

Could also have state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\Big(|2\rangle_1 \, |0\rangle_2 + |0\rangle_1 \, |2\rangle_2 \Big) \times |\text{vacuum}\rangle$$

= Superposition of two photons in mode 1 and two photons in mode 2

Neither mode in definite state by itself

If $\Psi \neq$ product of individual mode states say field is *entangled*

Interesting properties:

If detect one photon in mode 1:
State collapses to $|\Psi\rangle = |1\rangle_1 \times |\text{vacuum}\rangle$

Locate second photon without observing it!

Entangled states can violate Bell's Inequality
= proof that quantum mechanics is weird

May enable new technogies:
- quantum communication
- quantum computation

Could overcome limits of "classical" devices

Summary

- Photon optics:
  Light comes in packets, energy $\hbar\omega$
  Still progates as wave
- Quantum optics:
  Field itself becomes quantum variable
- “Normal” field = coherent state
  Exhibits quantum noise
  Other states possible
- Entangled states
  May have important applications