Phys 532 Midterm Solutions

1. Complex form

$$E = A\hat{j}\, e^{i(kz-\omega t)}$$

$$A = |A| e^{i\phi}$$

Real field $= |A|\hat{j}\cos(kz-\omega t+\phi)$

See $\hat{j} = \hat{x}$

If $E=0$ at $z=t=0$

need $\phi = \pm \pi/2$

If $E = 10\frac{V}{m}$ at $z = \frac{\lambda}{2}$, $t=0$

$= |A|\cos\left(\frac{\pi}{2} \pm \frac{\pi}{2}\right) > 0$

So want $-\frac{\pi}{2}$

and $|A| = 10\frac{V}{m}$

So $\boxed{E = \left(10\frac{V}{m}\right) e^{-i\pi/2}\, \hat{x}\, e^{i(kz-\omega t)}}$

2. From Hecht table 3.2, air has index 1.0003

For rare medium, $n-1 \propto N$ density $\propto$ pressure

So atop Everest, $\boxed{n \approx 1.0001}$

3. If AS reduced by 2, total power collected is reduced by 4.

If FS also reduced by two, fraction of collected power in image reduced by 4

So a) Total power in image $\boxed{\text{reduced by } 16}$

But $I_{image} = \dfrac{P_{image}}{A_{image}}$ reduced by 4, so

b) Irradiance $\boxed{\text{reduced by } 4}$

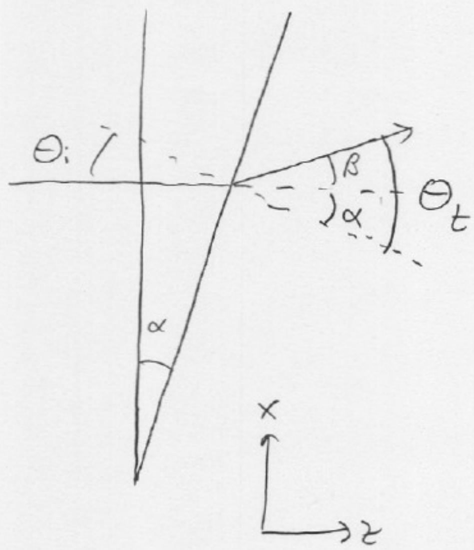
4. $NA = 0.05 \Rightarrow f/\# = 10$

So objective is $f/10 \Rightarrow$ $\boxed{\text{need a doublet}}$

But angle of rays into eyepiece is 10x smaller

So eyepiece is $f/100 \Rightarrow$ $\boxed{\text{use a singlet}}$

5. a) According to Snell's Law, the plane wave is deflected



Paraxial so

$$n\theta_i = \theta_t$$

$$\theta_i = \alpha$$

so $\theta_t = n\alpha$

Angle to z axis $= \beta = \theta_t - \alpha$

$$= (n-1)\alpha$$

Output is plane wave with

$$\vec{k} = k\,(\cos\beta\,\hat{z} + \sin\beta\,\hat{x})$$

But $\beta \ll 1$, so

$$\vec{k} = k\,(\hat{z} + \beta\hat{x})$$

Transmitted field is

$$\boxed{E_t = E_0\, e^{i[k(z+\beta x) - \omega t]}}$$

b) $E_t = E_{inc} + E_{scat}$

$$= E_{inc}\left(1 + i\,\frac{kL}{2}\,\chi\right)$$

$$= E_{inc}\left(1 + i\,\frac{k\alpha x}{2}\,\chi\right)$$

$$\boxed{E_t = E_0\, e^{i(kz - \omega t)}\left[1 + \frac{i}{2}\,\chi\, k\alpha x\right]}$$

(3½)

c) For small β, answer to (a) is

$$E_t = E_o e^{i(kz-\omega t)} \left[1 + ik\beta x\right]$$

$$= E_o e^{i(kz-\omega t)} \left[1 + ik\alpha(n-1)x\right]$$

But $n = \sqrt{1+\chi} \approx 1 + \frac{\chi}{2}$

So $n - 1 = \frac{\chi}{2}$ and

$$E_t = E_o e^{i(kz-\omega t)} \left[1 + i\frac{\chi}{2} k\alpha x\right]$$

$\boxed{\text{Same as (b)}}$

6. Use T to relate powers

$$T = \frac{n_t \cos\theta_i}{n_i \cos\theta_t} t^2$$

$$t_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

Have:
$P_{inc} = 10^5$ W
$I_{inc} = 3.3\times10^{10} \frac{W}{m^2}$
$u_{inc} = 111 \frac{J}{m^3}$
$E_{inc} = 5\times10^6 \frac{V}{m}$

$$\theta_i = 30^\circ$$

$$\sin\theta_i = n \sin\theta_t$$

$$\theta_t = 19.47^\circ$$

$$t_\perp = \frac{2\cos 30^\circ}{\cos 30^\circ + 1.5\cos 19.47^\circ} = 0.7596$$

$$T = \frac{1.5\cos 19.47^\circ}{\cos 30^\circ} (0.7596)^2 = 0.9422$$

So the energy of the transmitted pulse is

$$3\text{mJ} \times 0.9422 = \boxed{2.827\text{ mJ}}$$

$$I = \frac{\text{Power}}{\text{area}} \qquad \text{Power} = \frac{2.827\text{ mJ}}{30\text{ ns}} = 9.42\times10^4 \text{ W}$$

$$\text{Area} = 3\text{ mm}^2 \times \frac{\cos\theta_t}{\cos\theta_i}$$

$$= 3\text{ mm}^2 \times \frac{\cos 19.47^\circ}{\cos 30^\circ} = 3.27\text{ mm}^2$$

$$I_t = \frac{9.42\times10^4 \text{ W}}{3.27\text{ mm}^2} = \boxed{2.88\times10^{10} \frac{W}{m^2}}$$

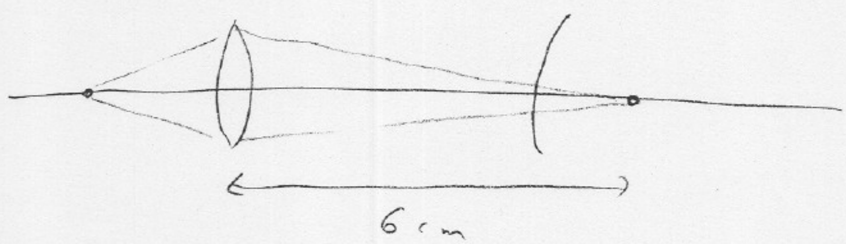
Have $I = \frac{n}{2Z_0} |E_0|^2$

$$E_0 = \sqrt{\frac{2Z_0}{n} I} = \sqrt{\frac{2 \cdot 377\,\Omega}{1.5} \cdot 2.88 \times 10^{10} \frac{W}{m^2}} = \boxed{3.81 \times 10^6 \frac{V}{m}}$$

7. Just use thin lens equation

Image from L1:

$$\frac{1}{S_i} = \frac{1}{f} - \frac{1}{S_o} = \frac{1}{2cm} - \frac{1}{3cm} = \frac{1}{6cm}$$



So for M1, have virtual object $S_o = -2cm$

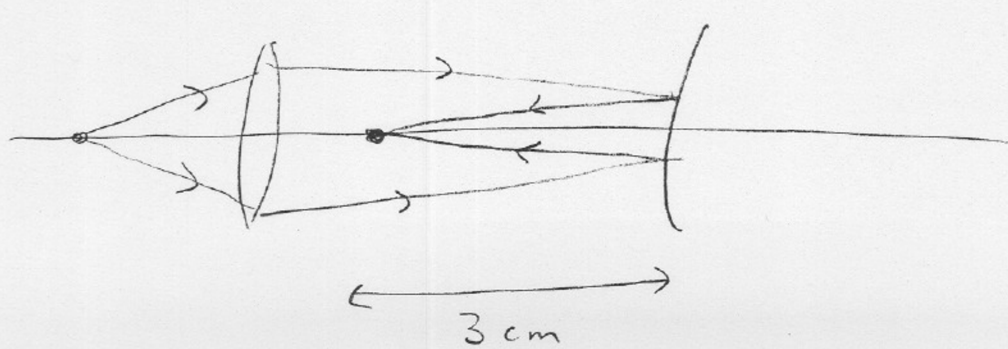
Acts like lens, focal length $= -\frac{R}{2} = -6cm$

So $\frac{1}{S_i} = \frac{1}{(-6cm)} - \frac{1}{(-2cm)} = \frac{1}{3cm}$

Real image... due to reflection,

image is in front of mirror

$\boxed{S_i = 3cm}$



Magnification:

For L1, $m = -\frac{s_i}{s_o} = -\frac{6\,cm}{3\,cm} = -2$

For M1 $m = -\frac{s_i}{s_o} = -\frac{3\,cm}{(-2\,cm)} = 1.5$

Total $m = (-2)(1.5) = \boxed{-3}$

8. a) $M_v = \begin{bmatrix} 1 & -\frac{1}{f} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2f & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{f} \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & -\frac{1}{f} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{f} \\ 2f & -1 \end{bmatrix}$

$$\boxed{M_v = \begin{bmatrix} -1 & 0 \\ 2f & -1 \end{bmatrix}}$$

b) $f_{sys} = -\frac{1}{B}$ $B = 0$ so $\boxed{f_{sys} = \infty}$

$ffl = bfl = \frac{-1}{0} = \infty$ also

Distance from vertex to principle plane

$$= \frac{A-1}{B} = \frac{-2}{B} = -\infty$$

So principle planes at $\boxed{\infty}$

c) From lecture notes, have

$$S_i = -\frac{c + d s_o}{a + b s_o}$$

$$= -\frac{2f - s_o}{-1} = 2f - s_o$$

$$\boxed{S_i = 2f - s_o}$$