Phys 531 Final Exam December 9 2:00-5:00 PM

Instructions:

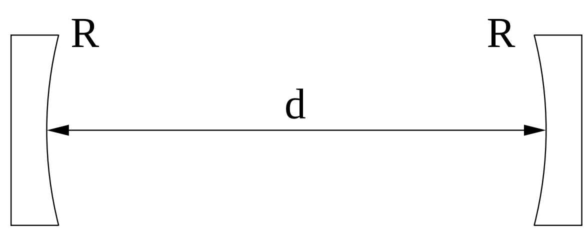
This is an in-class three-hour exam. You may refer to your textbook, lecture notes, and homework solutions. A calculator will be useful for some problems.

The exam consists of eight problems of varying difficulty. Each problem is worth 10 points and partial credit will be awarded. For maximum credit, be sure to explain your methods and write your solutions out clearly.

Some students are taking the exam late, so please be careful when discussing the exam that you don't inadvertently tell someone more than they should know.

1. Suppose a plane wave in vacuum has a wavelength of 480 nm.
(a) Give the oscillation frequency of the wave, both in rad/s and in Hz.
(b) If the wave enters a medium with index $n = 2$, what will be the new values for the wavelength, wavenumber, and oscillation frequency (in rad/s)?
(c) What color is this light?

2. An optical cavity can be constructed using two curved mirrors as shown. Cavities like this are essential elements of lasers. Here the radius of curvature of each mirror is $R < 0$ and the spacing between the mirrors is $d = -R/2$. Express your answers in terms of d.
(a) Calculate the ray matrix for light to make one complete round trip through the cavity. In other words, choose a starting position and calculate the total matrix for a ray to propagate to one mirror, reflect off that mirror, propagate to the other mirror, reflect again, and then propagate back to the starting point.
(b) Show that there is a unique Gaussian beam wave that occupies the cavity, such that after the beam makes a round trip through the cavity, it self-consistently returns to its initial value. Find the location and size of the beam waist, assuming a wavelength λ.



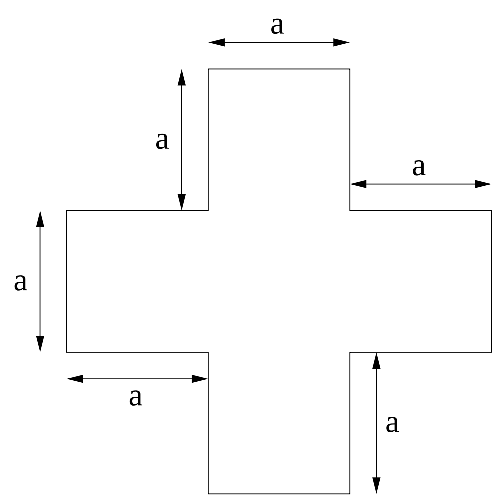
3. Suppose a wave with wavelength 500 nm is traveling in the $+z$ direction and has the values

$$E(x, y) = E_0 e^{i2\pi x/a} \cos(2\pi y/b)$$

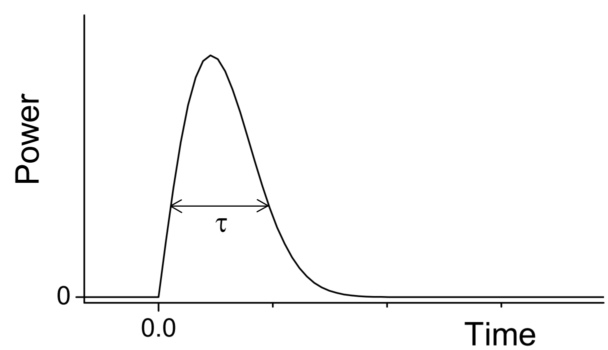
in the $z = 0$ plane, for $a = 1$ μm and $b = 2$ μm. Determine the values of the field $E(x, y, z)$ for all $z > 0$.

4. Calculate the Fraunhofer diffraction pattern produced when the "+" shaped aperture shown below is illuminated by a plane wave at normal incidence. (You can assume that the width a is large enough that the Fraunhofer approximation is valid.)

(Hint: there is a clever way to solve this that doesn't require you to do any hard integrals.)



5. Suppose a short pulse of light has central frequency ω_0 and duration τ. The figure shows a graph of the signal recorded when the pulse is measured with a power meter. Now suppose you put a bandpass filter in front of the power meter, such that the filter only transmits light within a narrow range of ω_0. Specifically, assume that the filter passes light in a range $\Delta\omega$ with $\tau\Delta\omega = 0.1$. Draw a sketch that shows (qualitatively) what the detected signal will look like with the filter in place.



6. Consider a parallel-plate interferometer made from a silicon wafer, which has an index of refraction $n = 3.6$ (and is transparent) for light with wavelength $\lambda = 1064$ nm. If a TE-polarized plane wave is incident at an angle near $\theta = 30°$, the irradiance of the transmitted light will vary as the angle is adjusted, due to interference. What is the minimum value of the transmission?

7. Suppose an object with a feature size of $a = 50$ μm is imaged with magnification 1 by an ideal lens with $f = 20$ cm and diameter $D = 2$ cm. In order to calculate the image field, should you use ray optics, the Fresnel approximation, or the Fraunhofer approximation? Assume light with wavelength 500 nm.

8. Suppose a material has different indices n_L and n_R for left- and right-circularly polarized light, respectively. If a horizontally polarized beam with wavenumber k passes through a length d of this material, what is the output polarization state? Simplify your result to standard form if necessary.