Phys 531 Assignment 11 Due December 6

1. Consider a Gaussian laser beam with wavelength $\lambda = 500$ nm that is bouncing back and forth between two highly reflective mirrors separated by a distance $d = 1$ cm. To prevent the beam from diverging as it propagates, assume that the mirrors are slightly curved and that the laser maintains a nearly constant beam waist $w_0 = 1$ mm. Since the reflecting beam forms a standing wave, the net intensity can be expressed as

$$I(\mathbf{r}) = I_0 e^{-2\rho^2/w_0^2} \cos^2 kz.$$

(You can be certain that $d = m\lambda/2$ for some large integer m.)

If the laser is so weak that there is only one photon on average in between the mirrors at a given time, what is the peak amplitude of the electric field in this region? Only consider the mean field from the beam itself, don't try to include any quantum noise effects.

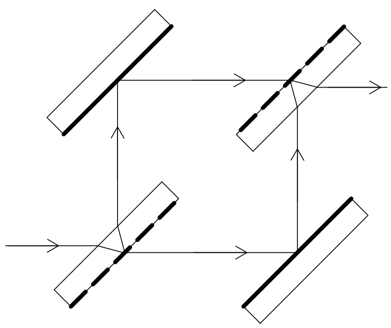
2. Suppose a linearly-polarized laser beam with wavelength 633 nm and power 1 mW passes through a quarter-wave plate and is converted to left-circular polarized light. The incident beam carries no angular momentum, while the output beam carries angular momentum along the direction of propagation. Calculate the resulting torque that the light exerts on the wave-plate.

(You don't need to answer this for the problem, but you might wonder: if the waveplate were free to rotate, it would evidently start to spin. Where does the energy for this motion come from?)

3. Consider a Mach-Zehnder interferometer as shown. If the input beam has average power P, the output beam will have power $P\cos^2\phi$ for interferometer phase ϕ. If you use such a device to measure ϕ over an averaging time T, then the uncertainty of the measurement will be

$$\Delta\phi = \frac{\Delta N}{|dN/d\phi|}$$

where N is the number of photons detected and ΔN is the measured noise. If the output is measured using a perfect photon counter and ΔN is limited only by quantum noise, what is the uncertainty $\Delta\phi$? What value of ϕ should be used to minimize $\Delta\phi$, and what is the minimum value obtained? The light has frequency ω and you can assume it to be in a coherent state.



4. An optical field is in a thermal state at temperature τ when it is in equilibrium with a thermal source. In this case, the probability to find n photons in the mode is $P_n = A\exp(-n\hbar\omega/k_B\tau)$, where ω is the mode frequency, k_B is Boltzmann's constant, and A is a normalization constant.

(a) Use the fact that $\sum P_n = 1$ to determine A and then calculate the photon noise

$$\Delta n \equiv \left(\langle n^2\rangle - \langle n\rangle^2\right)^{1/2}$$

for such a state.

(b) Determine $\Delta n/\langle n\rangle$ and sketch how it varies with τ.

(c) What are $\langle n\rangle$ and $\Delta n/\langle n\rangle$ for light with wavelength $\lambda = 500$ nm and $\tau = 6000$ K (the temperature of the surface of the sun).

The following results may be useful:

$$\sum_{n=0}^{\infty} x^{-n} = \frac{x}{x-1}$$

$$\sum_{n=0}^{\infty} nx^{-n} = \frac{x}{(x-1)^2}$$

$$\sum_{n=0}^{\infty} n^2x^{-n} = \frac{x(x+1)}{(x-1)^3}$$

(with $x > 1$ in all cases.)