Phys 531 Assignment 2 Due September 13

1. A light wave travels from point A to point B in vacuum, covering a distance of 1 cm. Suppose we introduce into its path a flat glass plate ($n_g = 1.50$) of thickness $L = 1.00$ mm. If the vacuum wavelength is 500 nm, how many wavelengths span the space from A to B with and without the glass in place? What is the total phase shift introduced with the insertion of the plate?

2. Suppose a laser produces a pulse of light with a duration of 1 ns, a diameter of 1 cm, and a total energy of 1 mJ. In free space, the pulse length is thus 30 cm, the energy density is 42 J/m^3, and the irradiance is 1.3×10^{10} W/m^2. Calculate the length, energy density and irradiance if the pulse is instead in a medium with $n = 1.5$. Compare the electric fields for the two cases.

3. In class we discussed a simple model for an atom, in which a cloud of negative charge (the electrons) surrounds a fixed nucleus of positive charge. Develop the details of this model as follows: Suppose an atom has one electron with charge $-e$ and a nucleus of charge $+e = 1.6 \times 10^{-19}$ C, and the electronic charge is uniformly distributed over a sphere with the Bohr radius $a_0 = 4\pi\epsilon_0\hbar^2/(me^2) = 5.29 \times 10^{-11}$ m. If the electron cloud is displaced a distance x, the magnitude of the attractive force between the cloud center and the nucleus is given by

$$F_{\text{Coulomb}} = \frac{1}{4\pi\epsilon_0}\frac{eq}{x^2}$$

where q is the portion of the electronic charge contained within a sphere of radius x. (This is easy to prove using Gauss's Law.)

(a) Using this result, show that the restoring force is linear in x and calculate the resulting oscillation frequency ω_0. The mass of the electron is 9.11×10^{-31} kg.

(b) Numerically compare your results to the Lyman-α resonance of hydrogen, which has a wavelength of 122 nm.

4. The optics catalog from the CVI Laser Corporation gives the index of refraction for fused silica (a type of glass) as a function of (vacuum) wavelength λ:

$$n^2(\lambda) = 1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3}$$

with $B_1 = 0.696$, $B_2 = 0.407$, $B_3 = 0.897$, $C_1 = 4.679 \times 10^{-3}$ μm^2, $C_2 = 1.351 \times 10^{-2}$ μm^2, and $C_3 = 9.793 \times 10^1$ μm^2.

(a) Explain how this formula is related to the atomic resonance model discussed in class (or alternatively in Hecht §3.5).

(b) The formula is only valid for a limited range of wavelengths. Based on the formula, what do you think is this range of validity? Also, for what wavelength range do you think fused silica is transparent?

5. Consider the scattered field produced by a cylinder of diameter $L \gg \lambda$, measured at a small but nonzero angle θ from directly forward. The picture shows a cross-section of the cylinder in the x-z plane. You can ignore any y-dependence in the problem.

We showed in class that the scattered field at $\mathbf{r} = (x_0, z_0)$ coming from a point $\mathbf{r}_j = (x_j, z_j)$ within the sphere is

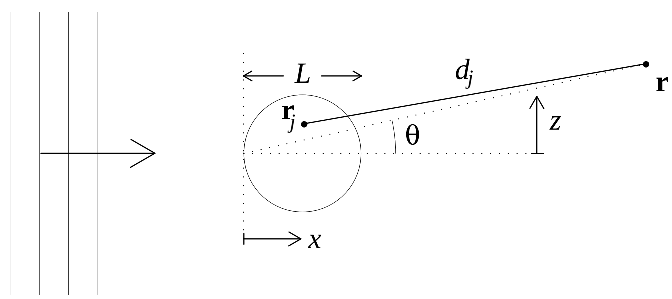
$$\mathbf{E}_j(\mathbf{r}) = \frac{k^2 \chi_1 E_0}{4\pi d} e^{i[k(x_j+d_j)-\omega t]}$$

where $d_j = |\mathbf{r} - \mathbf{r}_j|$ as shown. In order to have a large total scattered field at $\mathbf{r}$, the phases $\phi_j = k(x_j + d_j)$ from all the different points $\mathbf{r}_j$ must be the same to within a radian or so. Determine how large θ can be while meeting this criterion:

(a) Write down a formula for ϕ_j as a function of x_j, z_j, x_0, and z_0. Suppose that x_0 is very large and expand ϕ_j to first order in x_j and z_j.

(b) Determine the largest and smallest values that ϕ_j obtains as $\mathbf{r}_j$ varies. Calculate the difference $\Delta\phi$.

(c) Determine a bound on θ so that $\Delta\phi$ is less than 1.



6. Consider the path taken by light emitted by a source at point P, bouncing off a spherical mirror of radius R, and detected at point P'. The point P' is at the center of curvature of the mirror and P is a distance z away. The figure shows one possible path the light could take. Use Fermat's principle to find the true path and show that the true path represents a maximum length if P is to the right of P' (as shown), but a minimum length if P is to the left of P'.

(Hint: you can determine whether a function has a maximum or minimum at a point using its second derivative.)