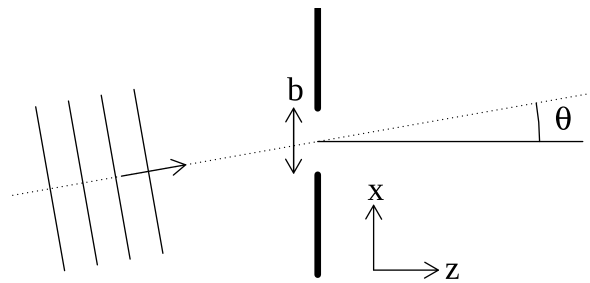
Phys 531 Assignment 8 Due November 3

1. Calculate the Fraunhofer diffraction pattern produced when a rectangular slit of width b and height L is illuminated by a plane wave making a small angle θ with respect to the plane of the slit, as shown. Show that the center of the diffraction pattern itself propagates at the same angle θ.



2. Consider an aperture consisting of two pinholes separated by a distance $2a$. Assume the pinholes are small enough to be approximated by δ-functions, so that

$$A(x,y) = [\delta(x-a) + \delta(x+a)]\delta(y)$$

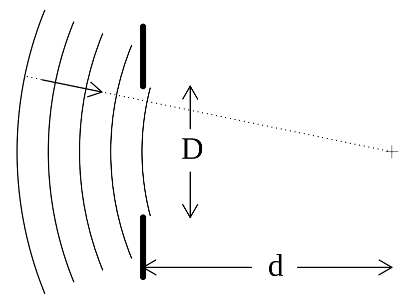
Calculate the diffracted field A_d produced at a distance d, using
(a) the Fresnel approximation
(b) the Fraunhofer approximation

3. Suppose an aberration-free lens is used to observe two point sources at infinite object distance. The two resulting images are a distance B apart. For large B, the points will be resolved and the image will show two distinct peaks. For small B, the points are not resolved and the image shows only one peak. In terms of the wavelength λ, the diameter of the lens D, and the focal length f, determine the value of B separating these two regimes. Find an answer accurate to within 2%:
(a) Assuming the two points are mutually coherent. For instance, they might be two tiny holes in a screen illuminated by a plane wave.
(b) Assuming the two points are mutually incoherent. For instance, they might be two distant stars (observed through a filter that transmits only light of wavelength λ). This prevents the two waves from interfering, so the observed image will be the sum of the individual irradiances.
Hint: A straightforward way to solve this is to use a computer program capable of plotting Bessel functions and see visually how close the image centers can get before merging into a single peak.

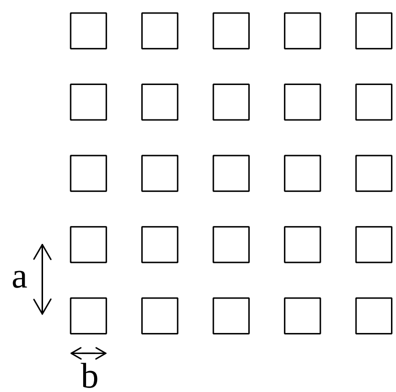
4. Suppose a converging spherical wave is incident on a circular aperture of diameter D, as shown. The field in the aperture is then

$$A(x,y) = E_0 \frac{e^{-ik\sqrt{x^2+y^2+d^2}}}{k\sqrt{x^2+y^2+d^2}}$$

Using the Fresnel approximation, calculate the resulting field $A_d(x,y)$ at $z=d$, the plane containing the center of the orginal wave. (If you can reduce your solution to the same form as an integral we did in lecture, you can just cite the lecture result, you don't need to recalculate it.)



5. Calculate the Fraunhofer diffraction pattern produced by a plane wave normally incident on a screen consisting of square holes with side b and center spacing a. There are a total of N^2 holes in the screen (in an $N \times N$ array).



6. Suppose a screen as in Problem 5 has $a = 0.5$ mm, $b = 0.1$ mm, and is illuminated with light having $\lambda = 500$ nm. The screen is the object of an imaging system as shown, where the lens focal length is $f = 100$ mm. A square hole with side p is placed in the focal plane of the lens so as to low-pass filter the image. What value of p should be used to give a smooth, uniform image with no modulation from the screen pattern? (You can assume here that $N \gg 1$.)