What is Light?

Historical discussion: Hecht Ch. 1

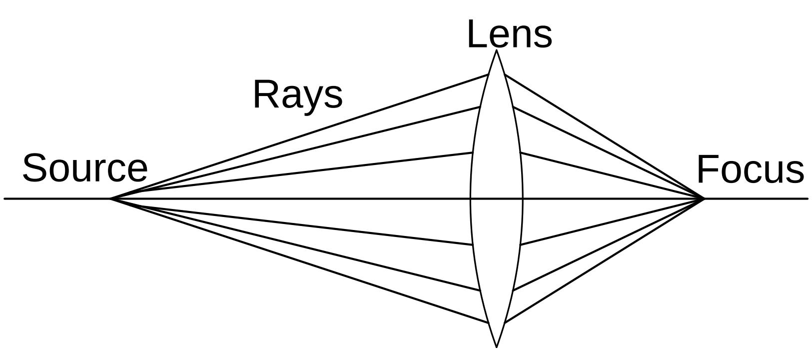
Simplest theory: light consists of a stream of particles.

The particles

- are emitted by a source (ie, a lamp),
- bounce off an object (ie, a book),
- and enter your eye.

Usually the particles travel in straight lines called *rays*.

Optical elements like lenses and mirrors work by deflecting the rays:



Particle theory is called ray optics or geometrical optics.

- The nature of the "particles" is not specified, so focus more on trajectories = rays.

Ray optics is useful for many problems in optics, including most imaging and illumination applications.

It fails to explain phenomena like interference, diffraction

- require wave optics

Wave Optics (Hecht Ch. 2)

Say light consists of a wave
= disturbance in a medium

Just like water waves, sound waves

For light, medium is "electromagnetic field"

- not very well defined, but doesn't matter

Wave optics is very accurate:
Treat wave optics as the "true" theory for most of this course

But ray optics is easier, use it when possible!

Quantum Optics

Wave optics still fails for some phenomena:
photoelectric effect, blackbody radiation

Best theory is quantum optics:
light has both wave and particle aspects

Really light is a quantum field
- trickier than "typical" quantum mechanics

Discuss a bit at end of course

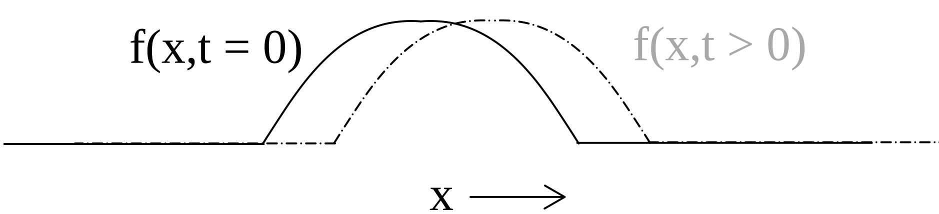
Waves (Hecht 2.1)

Start by thinking about how to describe waves.

Simple mathematical approach:
function describes a wave if

$$f(x,t) = f(x - vt)$$

Here f indicates the amplitude of the disturbance.
Shape of disturbance travels to $+x$ at speed v.



Call this a *travelling wave*.

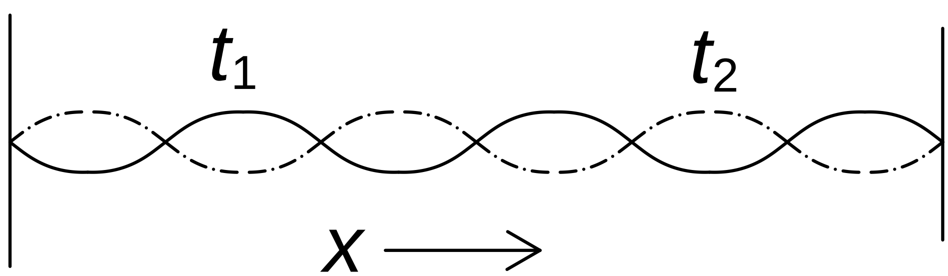
Three dimensional version: $f(\mathbf{r}, t) = f(\mathbf{r} - \mathbf{v}t)$
vector position $\mathbf{r}$, and velocity $\mathbf{v}$

Travelling waves easy to define, but too limiting.

- rock in pond: wave *spreads* in 2D
- sound wave: spreads in 3D

We'll see that these can be described by superpositions
= linear sums of travelling waves.

Simple example:
oscillating guitar string



Mathematically, $f(x,t) = \sin(kx)\sin(\omega t)$ for some k, ω.

Doesn't have form $f(x - vt)$, but still seems like a wave.

In fact, have $f(x,t) = \frac{1}{2}[\cos(kx - \omega t) - \cos(kx + \omega t)]$
= sum of two waves with $v = \omega/k$

Wave Equation

How can we tell if a function is a sum of travelling waves?

Any function $f(x - vt)$ has $\frac{\partial f}{\partial x} = -\frac{1}{v}\frac{\partial f}{\partial t}$.

Any function $f(x + vt)$ has $\frac{\partial f}{\partial x} = +\frac{1}{v}\frac{\partial f}{\partial t}$.

So if $\psi(x,t)$ is a sum of travelling waves, must have

$$\left(\frac{\partial}{\partial x} - \frac{1}{v}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x} + \frac{1}{v}\frac{\partial}{\partial t}\right)\psi(x,t) = 0$$

or

$$\frac{\partial^2\psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

Call this the wave equation. Say that function describes a wave if and only if it satisfies the wave equation.

Generalize to 3D:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

Recognize Laplacian operator $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
so write 3D wave equation as

$$\boxed{\nabla^2\psi(\mathbf{r},t) = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}}$$

Question: Which of these would you consider a wave?

Dye spreading in a pool of water.

A line of dominos falling over.

A superposition of two travelling waves with different speeds.

Which do you think satisfies the wave equation?

Harmonic Waves (Hecht 2.2)

Most important solutions of wave equation are harmonic waves:

$$\psi(x,t) = A\cos(kx - \omega t + \phi)$$

where

$A \equiv$ amplitude
$k \equiv$ wave number (units rad/m, usually just m^{-1})
$\omega \equiv$ frequency (units rad/s)
$\phi \equiv$ phase (units rad)

Also have

$\lambda = 2\pi/k \equiv$ wave length (units m)
$\tau = 2\pi/\omega \equiv$ period (units s)
$\nu = 1/\tau = \omega/2\pi \equiv$ frequency (units cycles/s or Hz)

Also, the wave equation requires that $\omega = vk$.

Harmonic waves are periodic in both space and time:

$$\psi(x+\lambda,t) = \psi(x,t+\tau) = \psi(x,t)$$

These definitions and relationships are very important, so you should memorize them!

Question: What are the units of A?

The 3D version of a harmonic wave is called a *plane wave*:

$$\psi(\mathbf{r},t) = A\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$$

Here $\mathbf{k}$ is the wave vector, while $k = |\mathbf{k}|$ is the wave number.

We still have $k = \omega/v = 2\pi/\lambda$. The condition of spatial periodicity becomes

$$\psi(\mathbf{r} + \lambda\widehat{\mathbf{k}}, t) = \psi(\mathbf{r}, t)$$

where $\widehat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|$ is the propagation direction of the wave.

Complex Representation (Hecht 2.5, handout)

Harmonics waves are useful, but trig functions get tedious. Instead represent with complex functions.

Complex numbers: form $z = x + iy$, where $i = \sqrt{-1}$.
Define $x \equiv$ real part, write Re z
$y \equiv$ imaginary part Im z.

Complex numbers follow the normal rules of algebra:

$$(x_1 + iy_1) + (x_2 + iy_2) = (x_1 + x_2) + i(y_1 + y_2)$$

$$\begin{aligned}(x_1 + iy_1)(x_2 + iy_2) &= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2) \\ &= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 \\ &= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)\end{aligned}$$

Also define complex conjugate $z^* \equiv x - iy$
magnitude $|z| \equiv \sqrt{x^2 + y^2} = \sqrt{zz^*}$

Main reason we use complex numbers is Euler identity:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Prove with Taylor's expansions:

$$\begin{aligned} e^{i\theta} &= 1 + i\theta + \frac{1}{2}(i\theta)^2 + \frac{1}{6}(i\theta)^3 + \ldots \\ &= \left(1 - \frac{1}{2}\theta^2 + \ldots\right) + i(\theta - \frac{1}{6}\theta^3 + \ldots) \\ &= \cos\theta + i\sin\theta \end{aligned}$$

So can write any complex number in *polar form*: $z = re^{i\theta}$
with $r\cos\theta = x$ and $r\sin\theta = y$
Note $|z| = r|e^{i\theta}| = r$

Use Euler identity to write a harmonic wave as

$$\psi(x,t) = \text{Re } \left\{\left[Ae^{i\phi}\right] e^{i(kx-\omega t)}\right\}$$

Usually just write

$$\psi(x,t) = Ae^{i(kx-\omega t)}$$

where

- $A = |A|e^{i\phi}$ is complex: called complex amplitude
- implicit that only real part of ψ is actual wave

This lets us work with exponentials instead of sines and cosines.

Do all math with complex form, take real part at end.

Question: What makes exponentials easier to use than trig functions?

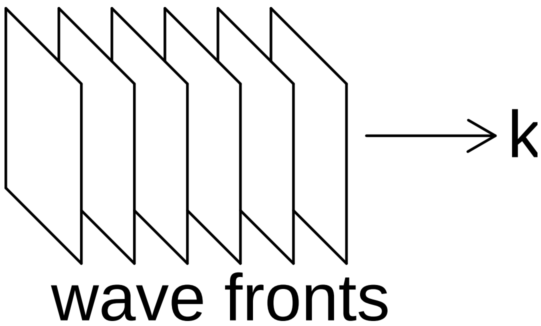
Generalize to 3D:

Plane wave $\psi(\mathbf{r},t) = Ae^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

Called plane wave because surfaces of constant ψ are planes

- surfaces called *wave fronts*

Here wavefronts $\perp \hat{\mathbf{k}}$



Spherical wave (Hecht 2.9)

Another 3D wave:

$$\psi = \frac{A}{r}e^{i(kr-\omega t)}$$

- $A =$ (complex) amplitude
- $r = |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$
- Wave fronts are spheres centered at $r = 0$
- Represents wave expanding from point source

Converging spherical wave: $\psi = \frac{A}{r}e^{i(kr+\omega t)}$

Use spherical waves for light emitted by source or converging to focus.

Plane Wave Decomposition

Can write spherical wave as sum of plane waves:

$$\frac{e^{ikr}}{kr} = \frac{i}{2\pi} \iint_{-\infty}^{\infty} \frac{1}{m} e^{ik(px+qy+m|z|)} dp\, dq$$

for $m = \sqrt{1 - p^2 - q^2}$ (can be imaginary)

This is the Weyl representation of a spherical wave

- Uses complex wave vectors
- Actually pretty hard to prove
- Demonstrates point about nontrivial superpositions

Summary

- Light is a wave
- "Wave" defined by wave equation
- Superposition of waves = new wave
- Plane waves very important

  = 3D harmonic travelling waves