Phys 531 Lecture 11 29 September 2005

Survey of Optical Systems

Last time:

Developed tools to analyze optical systems

- ray matrix technique
- thick lens picture

Today, look at several common systems

Won't use matrix methods explicitly

but many lenses thick:

implicitly use thick lens picture

Outline:

- the eye
- eye glasses
- magnifying glass
- microscope

These and more examples: Hecht 5.7

Next time:

advice for designing systems of your own

The Eye (Hecht 5.7.1)

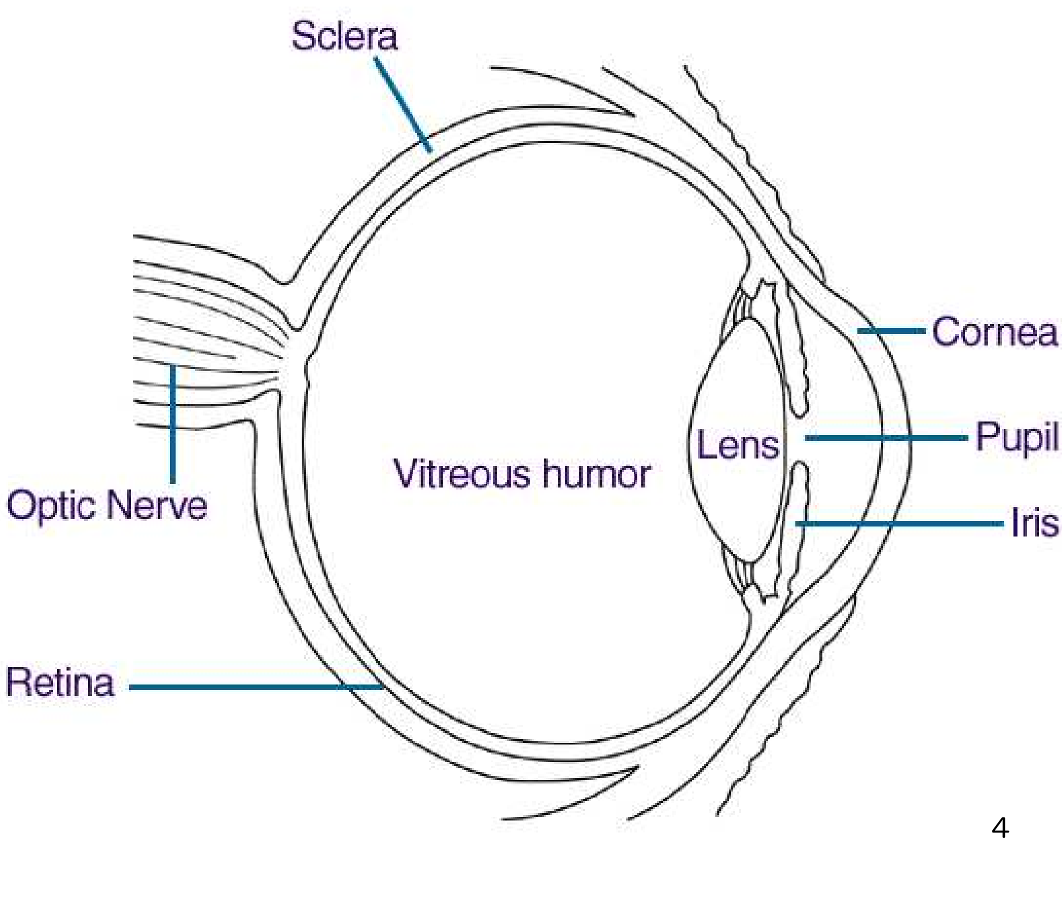
Most basic optical system

Components:

- cornea: $n = 1.376$
- vitreous humor $\approx$ water: $n = 1.33$
- lens: $n = 1.39 - 1.41$
  index varies: high in center, low at edges
- iris: variable aperture stop
  diameter 2-8 mm
- retina: detector

Again, use thick lens picture

lens system has well defined focal length



Note : detector is in medium $n_i = 1.33$

Lens equation becomes

$$\frac{1}{s_o} + \frac{n_i}{s_i} = \frac{1}{f_o} = \frac{n_i}{f_i}$$

f_o = object focal length
f_i = image focal length

Irrelevant for ray matrix
Modifies thick lens picture:

f_o applies to front focal point
f_i applies to back focal point

System focal length variable

max object distance = ∞ ("relaxed")
min object distance $\approx$ 25 cm (varies)

What focal lengths?
Distance from lens to retina $\approx$ 24 mm $\approx s_i$

Relaxed eye: $s_o = \infty \Rightarrow f_i = s_i$

For $s_o = 25$ cm: $\frac{n_i}{f_i} = \frac{1}{s_o} + \frac{n_i}{s_i} \Rightarrow f_i = 22$ mm

So $f_i = 22 - 24$ mm
Called "accommodation"

Most focusing power from cornea-air interface

$$f_i = \frac{n}{n-1} R$$

$R \approx 9$ mm $\Rightarrow f_i \approx 33$ mm

Remaining surfaces:

$$\frac{1}{f_{\text{total}}} \approx \frac{1}{f_{\text{cornea}}} + \frac{1}{f_{\text{rest}}}$$

So

$$\frac{1}{f_{\text{rest}}} = \frac{1}{24} - \frac{1}{33} = \frac{1}{88 \text{ mm}}$$

for relaxed eye

$f_{\text{rest}} = 66$ mm for accommodated eye

f_{rest} adjusted by squeezing lens

muscles relaxed: f long

muscles tense: f short

Minimum achievable s_o = *near point*

depends on flexibility of lens

varies with age

Question: Why can't you see well under water?

Capabilities of the eye

Resolution:

angular resolution $\Delta\theta \approx 0.017^\circ = 0.3$ mrad

Just adequate to resolve crescent of Venus

Correpsonds to about 5 μm on retina

At $s_o = 25$ cm,
spatial resolution $= s_o\Delta\theta = 75\ \mu$m

Also, wide field of view:

corresponds to 100 Mpixels!

Resolution best in center

Sensitivity:

Fully expanded pupil, can see $I \leq 10^{-10}$ W/m^2 from point source

Power $= IA$
Area $= \pi(4\text{ mm})^2 \Rightarrow P \approx 10^{-14}$ W

Maximum irradiance:

sunlight $I \approx 250$ W/m^2

pupil area $\pi(1\text{ mm})^2$

Max power $= 10^{-3}$ W

But: sun is not point source

power spread out on retina

Sun subtends angle 10 mrad $\approx 30 \times \Delta\theta$

Same intensity from point source:

illuminate area $30^2\times$ smaller on retina

$\approx 1000\times$ higher image irradiance

Max power from point source $\approx 10^{-6}$ W

($\approx$ damage threshold for laser)

Dynamic range of eye: 10^{-14} to 10^{-6} W

eight orders of magnitude

Instantaneous range lower:

$\sim$ five orders of magnitude

Best artificial detectors:

photographic film

high-end CCDs

dynamic range $\approx$ four orders of magnitude

$10\times$ worse than eye

Upshot:

Can't build a detector nearly as good as the eye

Eyeglasses (Hecht 5.7.2)

Common problem: focal length of eye isn't right

Too strong = near sighted = myopic:

relaxed eye has $f_i < 24$ mm

$$\frac{1}{s_{o\mathrm{max}}} = \frac{n}{f_i} - \frac{n}{s_i} > 0$$

so can't focus at ∞

Maximum distance of focus = *far point*

Easy to measure

For me, far point ≈ 25 cm

Also moves near point closer:

for me $s_{o\mathrm{min}} \approx 7$ cm

What is my range of f? (assuming $s_i = 24$ mm)

$$\frac{1}{f_{\mathrm{min}}} = \frac{1}{1.33 \cdot 70 \text{ mm}} + \frac{1}{24 \text{ mm}}$$

$$\Rightarrow f_{\mathrm{min}} = 19 \text{ mm}$$

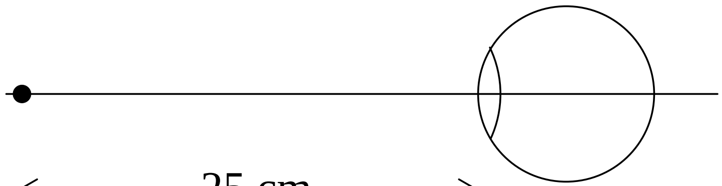
and

$$\frac{1}{f_{\mathrm{max}}} = \frac{1}{1.33 \cdot 250 \text{ mm}} + \frac{1}{24 \text{ mm}}$$

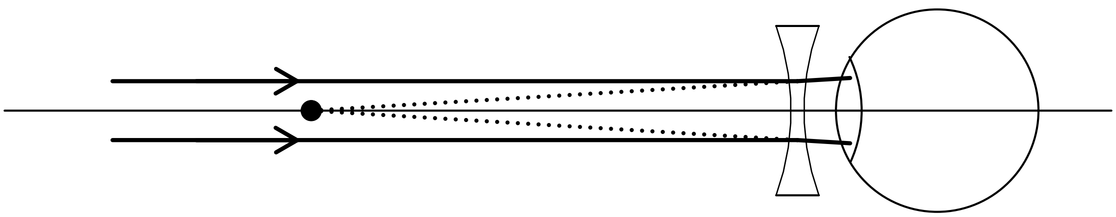
$$\Rightarrow f_{\mathrm{max}} = 22 \text{ mm}$$

Fix with eye glasses

Relaxed eye:



Add lens to put image of ∞ at 25 cm:



What focal length required?

want $s_o = \infty$, $s_i = -25$ cm

So $f = -25$ cm

This is my prescription:

$$\mathcal{D} = \frac{1}{f} = -4 \text{ diopters}$$

How close is near point with glasses on?

s_o such that s_i = -7 cm for $f = -25$ cm

$$\frac{1}{s_o} = -\frac{1}{25} + \frac{1}{7} = \frac{1}{10 \text{ cm}}$$

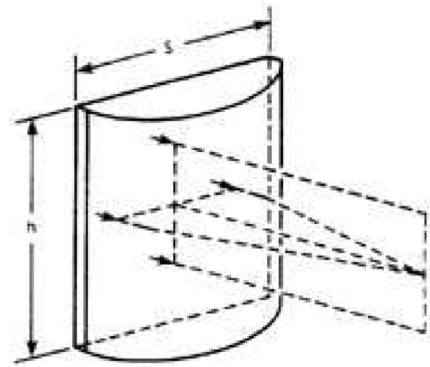
Other vision problems:

Far sighted = hyperopia:
    eye's lens too weak

Correct with positive lens

Astigmatism:
asymmetry in lens
    f's different along x, y

Correct with cylindrical lens



Question: If you want to start a fire with your glasses, should you be near-sighted or far-sighted?

Magnifying glass (Hecht 5.7.3)

At 25 cm, typical eye can resolve 75 μm

Use a lens to see something smaller...
    what kind?

Want erect, magnified image of real object

- Real object: $s_o > 0$
- Erect: $m = -s_i/s_o > 0$ so $s_i < 0$
- Magnified: $m > 1$ so $|s_i| > |s_o|$

Have

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Want $1/s_o$ positive and large

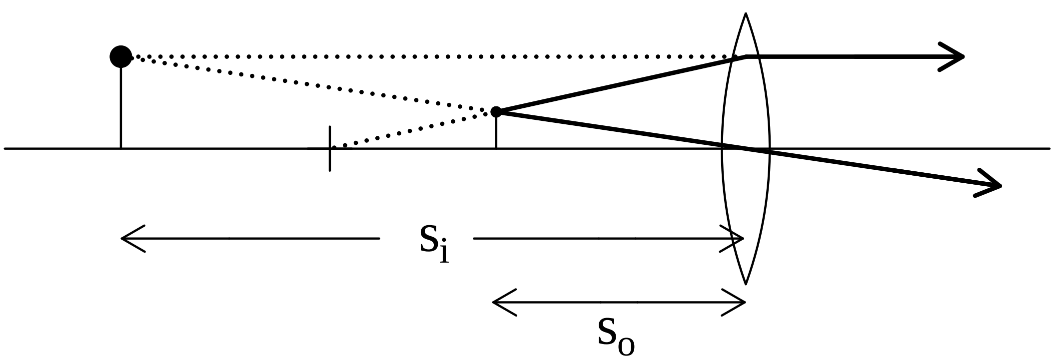
$1/s_i$ negative and small

Means f should be positive

Recall: get virtual image with positive lens

when $s_o < f$

Picture:

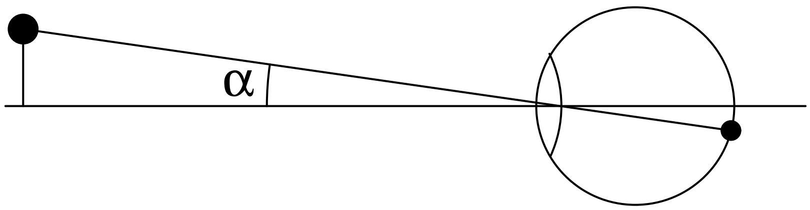


See image is magnified

But also further away...

Resolution improved if image on retina is bigger

Note size of image on retina proportional to angular size of object



Size on retina = αf

Don't really know f, just consider α

Aside:

Define *magnifying power* of system (MP)

$= $ angular magnification

$$= \frac{\alpha \text{ with lens}}{\alpha \text{ without lens}}$$

Write as MP = 5×, etc.

Could make α without lens very big:

hold object right up to eye

But can't focus if $s_o <$ near point

For magnifying glass, microscope, etc
(not telescope)
Define α for standard distance $s_o = 25$ cm

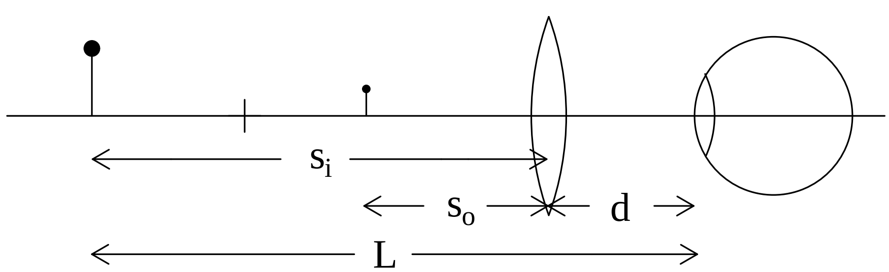
Example:
If I take off my glasses, near point is 7 cm

Object at 7 cm subtends $\alpha = y/7$ cm
Object at standard 25 cm subtends $\alpha_0 = y/25$ cm

$$\text{Magnifying power } = \frac{\alpha}{\alpha_0} = \frac{25}{7} = 3.6$$

My bare eyes have MP = $3.6\times$

What is MP of glass?



Express in terms of practical parameters
d = distance from eye to glass
L = distance from eye to image
f = focal length of glass

Have object size y_o
image size y_i

Angular size of image $\alpha = \frac{y_i}{L} = \frac{my_o}{L}$

magnification $m = -s_i/s_o$:

$$\alpha = -\frac{s_i}{s_o}\frac{y_o}{L}$$

Without glass $\alpha_0 = \frac{y_o}{d_o}$

$$\text{So MP} = \frac{\alpha}{\alpha_0} = -\frac{s_i}{s_o}\frac{d_o}{L}$$

Eliminate s_o, s_i:

Have $s_i = d - L$

and $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$

So $\frac{s_i}{s_o} = \frac{s_i}{f} - 1 = \frac{d - L}{f} - 1$

Gives $\boxed{MP = \left(1 + \frac{L - d}{f}\right)\frac{d_0}{L}}$

Two reasonable ways to use:

• Make $L \to \infty$

Achieve by making $s_o \to f$ (so $s_i \to \infty$)

View image with relaxed eye, d doesn't matter

Get MP $\to \dfrac{d_0}{f}$

Large MP for small f

Other method:

• Lens close to eye $d \to 0$

$$\text{MP} \to d_0 \left(\frac{1}{L} + \frac{1}{f} \right)$$

To get large MP, want L small
minimum L = near point = d_0

Then MP $\to 1 + \dfrac{d_0}{f}$

Large if f is small

Where do we need to put object?

$$
\begin{aligned}
\frac{1}{s_o} &= \frac{1}{f} - \frac{1}{s_i} \\
&= \frac{1}{f} + \frac{1}{d_0} \\
&= \frac{1}{d_0}\left(1 + \frac{d_0}{f}\right)
\end{aligned}
$$

So $s_o = \dfrac{d_0}{\mathrm{MP}}$

Recall eye example: MP $= d_0/s_o$ . . . same

Object looks as it would if you could focus at s_o

Lens makes eye stronger

like being near-sighted

Works well if you can hold object up to eye

Either method works up to about 4×
f down to 6 cm

For higher MP, not paraxial:
- lens aberrations important
- requires more complex lens

Still works:
Method 2: Jeweler's loupe
Get MP up to 30×

Impractical if object position fixed
or MP so high you can't hold steady

Already a problem at 10×

Method 1: put lens very close to object
since $s_o \approx f$ and f is small

Problem: exit pupil is small and far away
- can't see very much

Question: Where is the exit pupil in this case?

Solution: compound microscope

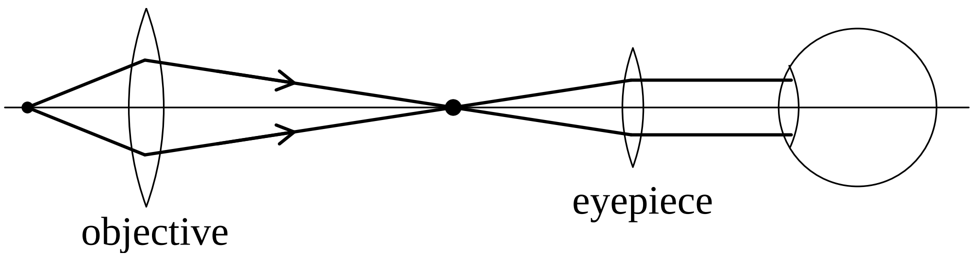
Microscope (Hecht 5.7.5)

Use two lenses:

objective: short f, close to object

eyepiece: collects light, match to eye pupil

Typical system:



Typically don't care about inversion:

- object creates real inverted image

Objective collects rays at steep angles:

- important to control aberrations

Eyepiece:

- puts final image at ∞
  - view with relaxed eye
- provides additional magnification
- matches exit pupil to eye

Aberrations less important than in objective

What is magnifying power?

Objective: magnification $= -s_i/s_o$
Angular magnification not appropriate:
    intermediate image not viewed by eye

Even so, there is a standard length scale

Want objectives interchangeable:
    standard position for object, image

Set by *tube length*
    = distance from back focal point to image
    = 160 mm

Then $s_i = 160$ mm $+ f$

$$\frac{1}{s_o} = \frac{1}{f} - \frac{1}{s_i}$$

$$\text{MP} = \frac{s_i}{s_o} = \frac{s_i}{f} - 1$$

$$= \frac{160 \text{ mm} + f}{f} - 1 = \frac{160 \text{ mm}}{f}$$

This is magnification written on objective

So 20× objective has $f = 8$ mm

$$\text{Total MP} = \text{MP}_{\text{obj}} \cdot \text{MP}_{\text{eyepiece}}$$
$$= \frac{160 \text{ mm}}{f_{\text{obj}}} \cdot \frac{250 \text{ mm}}{f_{\text{eyepiece}}}$$

where MP for eyepiece follows standard convention

Typical objective: 5× to 60×

Typical eyepiece: 5× or 10×

Warning:
Fancy microscopes don't follow these conventions

Summary:

- Eyes are impressive instruments, both for sensitivity and resolution
- Eyeglasses/contacts work by adjusting location of near and far points
- Effect of magnifying glass is really *angular* magnification
  - Measure with magnifying power
- Microscope uses two lenses to provide more MP than glass