Phys 531 Lecture 8 20 September 2005

Ray Optics I

Last time, finished EM theory

Looked at complex boundary problems

TIR: Snell's law complex

Metal mirrors: index complex

Today shift gears, start applying theory

want to manipulate light

Study how lenses, mirrors, etc. work

and how they work together in a system

For next five lectures, focus on *ray optics*

= "particle" theory of light

Simpler approximation to wave theory

Outline:

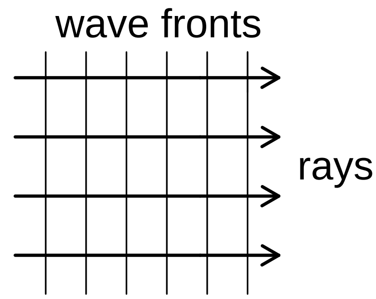
- Ray optics
- Ideal imaging surfaces
- Paraxial optics
- Thin lenses

Next time: finish lenses,
cover mirrors, prisms, apertures

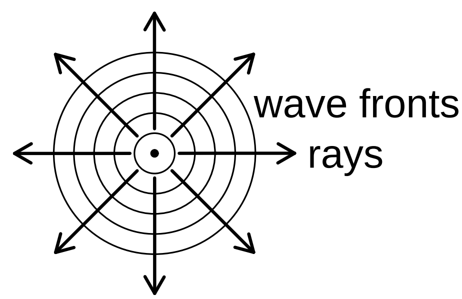
Ray Optics

Formally, ray = vector normal to wave front
draw as line through space

Plane wave: ray = $\hat{\mathbf{k}}$

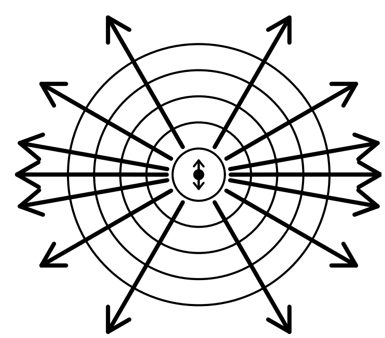


Spherical wave:
rays point into or
out of source



Sometimes use density of rays to indicate intensity

Dipole radiation:

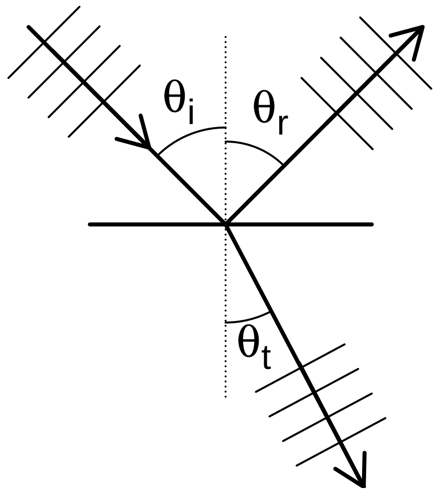


Wave defined by wavefronts
equivalently by rays

In free space, rays are straight

At boundaries, Snell's law, law of reflection describe what $\hat{\mathbf{k}}$ does

= describe what rays do

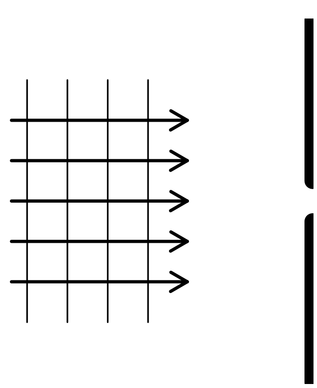


Ray optics: interpret rays as trajectories of particles

Leads to incorrect predictions:

- No interference
- Gets trajectories wrong

Example: absorbing sheet with small hole



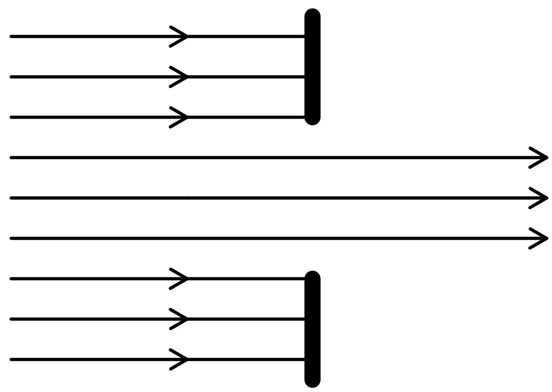
Ray optics:

predict particles entering hole

continue undisturbed

other particles blocked

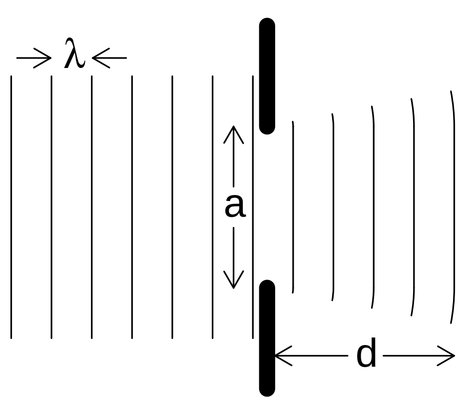
Expect thin pencil of light transmitted



Wave optics:

don't (yet) know how to predict

Will find transmitted wave diverges: diffraction



Divergence important for $d \gtrsim \frac{a^2}{\lambda}$

a = hole size $\qquad$ d = propagation distance

Generally, ray optics valid when

(a) no (explicit) interference

(b) feature size $>$ (propagation distance $\times \lambda)^{1/2}$

For $\lambda \approx 1$ μm, $d \approx 1$ m, need $a > 1$ mm

Rule of thumb:
ray optics OK for elements larger than 1 mm

(element = lens, mirror, aperture, etc.)

Question: A laser beam is often considered as a pencil of rays. If a beam has diameter 1 cm and wavelength 1 μm, over what propagation distance is ray optics valid?

Lenses (Hecht 5.2)

For now, assume ray optics is valid

Two main applications:

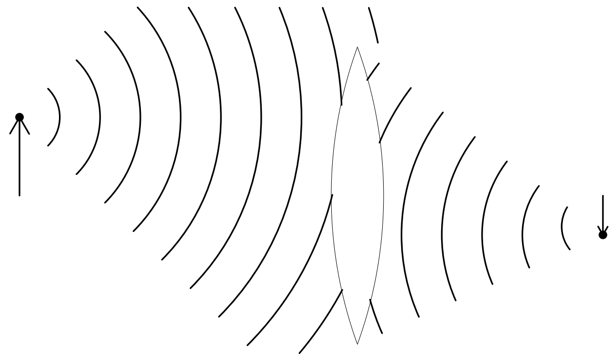
- Image formation: have an object, want to take its picture
- Illumination: have a source, want to direct light to target

Both require light directed from place to place

Basic tools: lenses, mirrors, prisms

Start with lenses

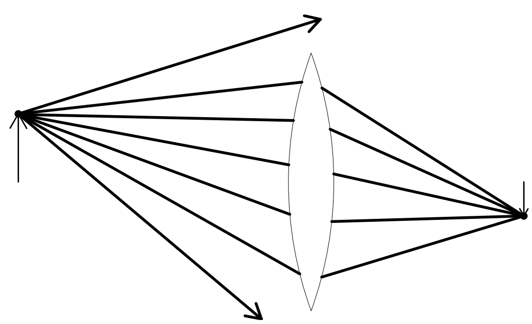
Lens = curved refracting surface or surfaces
used to change center of spherical wave



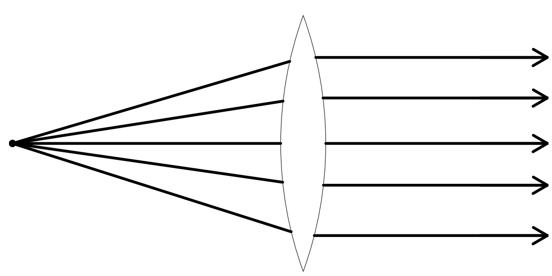
Each point on object emits spherical wave

Lens makes wave converge to new point

Ray optics: lens *focuses* set of rays to a point

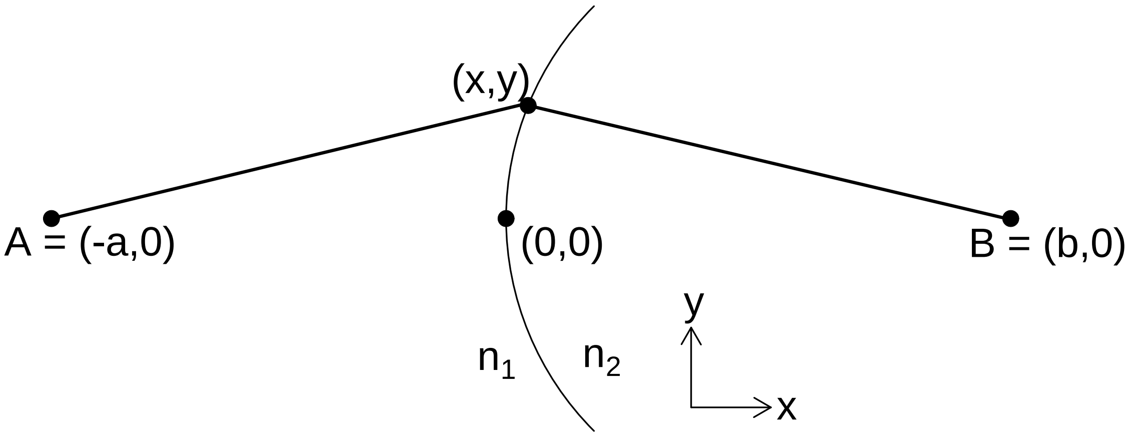


Point $\rightarrow \infty$: *collimate rays* = make parallel



What shape should surface have?

Consider single surface $n_1 \rightarrow n_2$



Surface defined by points $y = f(x)$

want to determine right function f

Want all rays from A to reach B

Fermat's principle:
all paths from A to B have same $\mathcal{S}$

Path going through (x, y):

$$\mathcal{S} = n_1\sqrt{(x+a)^2 + y^2} + n_2\sqrt{(x-b)^2 + y^2}$$

Know for point $(0, 0)$: $\mathcal{S} = n_1 a + n_2 b$

If $\mathcal{S}$ constant, then

$$n_1\sqrt{(x+a)^2+y^2}+n_2\sqrt{(x-b)^2+y^2}=n_1a+n_2b$$

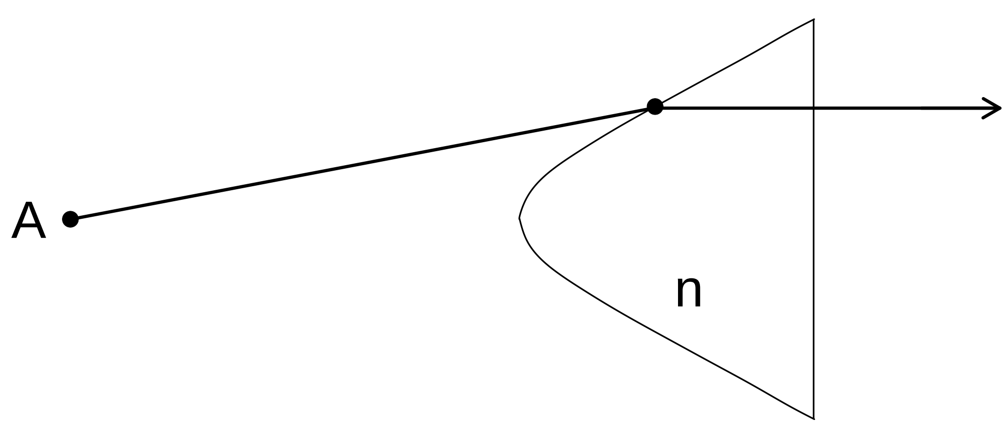
In principle, solve for $y=f(x)$

Example: $b\to\infty$, $n_2>n_1$

get $y=\dfrac{1}{n_1}\sqrt{2an_1(n_2-n_1)x+(n_2^2-n_1^2)x^2}$

Can show this is equation for hyperbola
(hyperboloid in 3D)

If you don't want image in medium,
need second surface



Generally use sphere centered at B:
doesn't deflect rays

This technique gives "ideal" lens
all rays hitting lens reach B

Unfortunately, ideal lens hard to construct

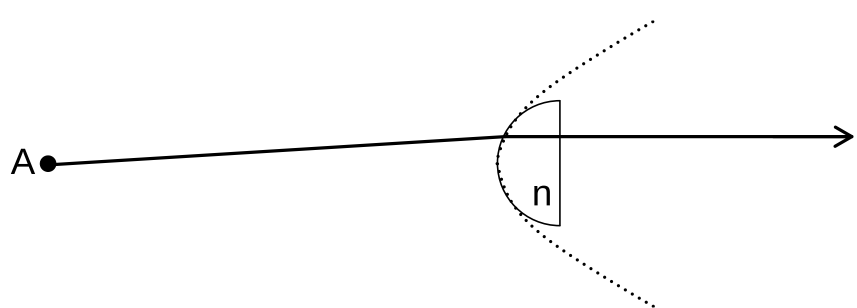
Require surface accuracy $\sim \lambda/4$,
otherwise waves don't add constructively

Also, limited to particular points A and B

Usually have extended object:
many source and image points

Can make one kind of surface precisely:
sphere

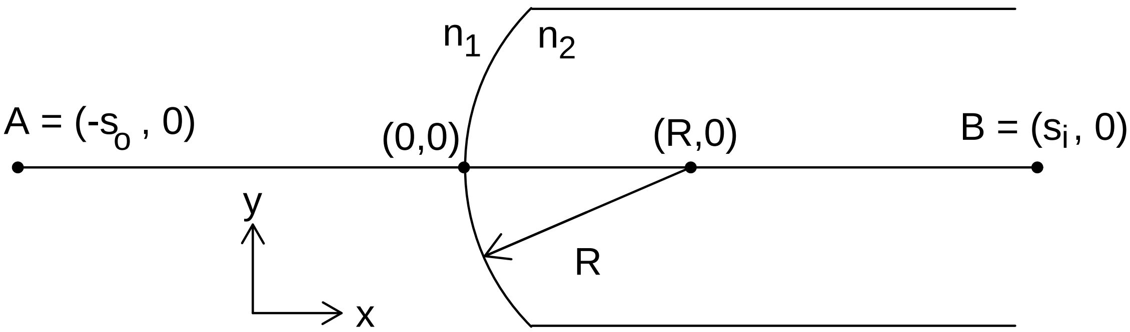
Strategy: approximate ideal surface by sphere



OK if y small enough

(Ideal lens usually called aspheric)

Try to design spherical lens



Find R such that $\mathcal{S}$ from A to B constant
for small y

($s_o =$ "object distance;" $s_i =$ "image distance")

Surface is sphere centered at $(R, 0)$

$$y^2 + (x - R)^2 = R^2$$

$$y^2 = 2xR - x^2$$

So $\mathcal{S} = n_1\sqrt{(x + s_o)^2 + y^2} + n_2\sqrt{(x - s_i)^2 + y^2}$

$$= n_1\sqrt{(x + s_o)^2 + 2xR - x^2}$$

$$+ n_2\sqrt{(x - s_i)^2 + 2xR - x^2}$$

$$= n_1\sqrt{s_o^2 + 2x(R + s_o)} + n_2\sqrt{s_i^2 + 2x(R - s_i)}$$

Want $\mathcal{S}$ constant: $\frac{d\mathcal{S}}{dx} = 0$

$$\frac{d\mathcal{S}}{dx} = \frac{n_1(R + s_o)}{\sqrt{s_o^2 + 2x(R + s_o)}} + \frac{n_2(R - s_i)}{\sqrt{s_i^2 + 2x(R - s_i)}}$$

No solution in general, but we want small y

$\Rightarrow$ very small x

$$\left(\text{if } y \ll R \text{ then } x \ll \frac{y^2}{2R}\right)$$

So set $x = 0$:

$$\left.\frac{d\mathcal{S}}{dx}\right|_{x=0} = n_1 \frac{R + s_o}{s_o} + n_2 \frac{R - s_i}{s_i}$$

So want

$$\begin{aligned} 0 &= n_1 \frac{R + s_o}{s_o} + n_2 \frac{R - s_i}{s_i} \\ &= n_1 \left(\frac{R}{s_o} + 1\right) + n_2 \left(\frac{R}{s_i} - 1\right) \\ &= \frac{n_1}{s_o} + \frac{n_1}{R} + \frac{n_2}{s_i} - \frac{n_2}{R} \end{aligned}$$

or

$$\boxed{\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}}$$

Relates R, s_o and s_i: know two, solve for other

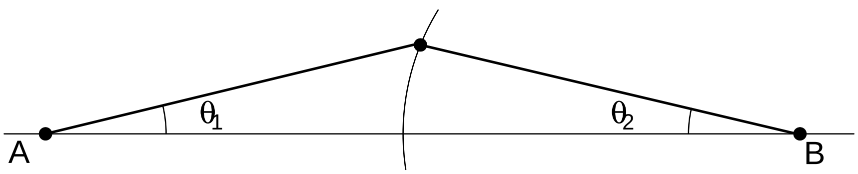
Spherical lens works for rays with $y \ll R$

but $R = \dfrac{n_2 - n_1}{\left(\frac{n_1}{s_o} + \frac{n_2}{s_i}\right)}$

so $y \ll \dfrac{(n_2 - n_1) s_o s_i}{n_1 s_i + n_2 s_o} \approx \dfrac{s_o s_i}{s_o + s_i} \approx \min(s_o, s_i)$

Unless $n_1 \approx n_2$, need $y \ll s_o$ and $y \ll s_i$

Have $y_1/s_o \approx \theta_1$ and $y_1/s_i \approx \theta_2$



Best statement:
spherical lens works for rays with $\theta \ll 1$

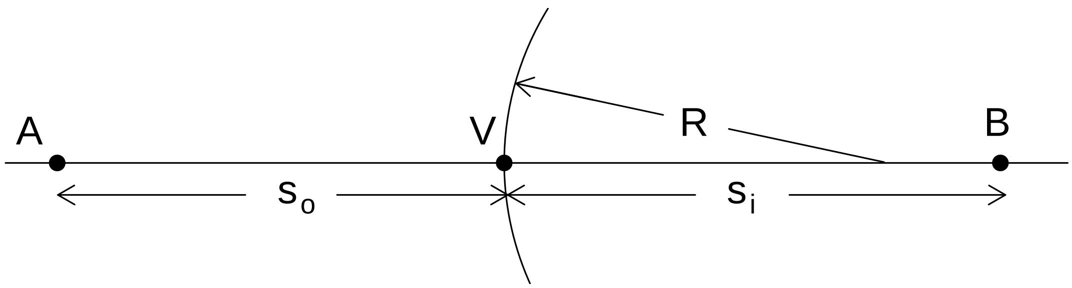
Called *paraxial rays*

Lens formula equivalent to approximation $\sin\theta \approx \theta$
in Snell's law

Treatment of lenses with paraxial rays:
Gaussian, paraxial, or first-order optics

Deviations from paraxial give aberrations
= imaging errors

Conventions and Definitions
(Hecht Table 5.1)

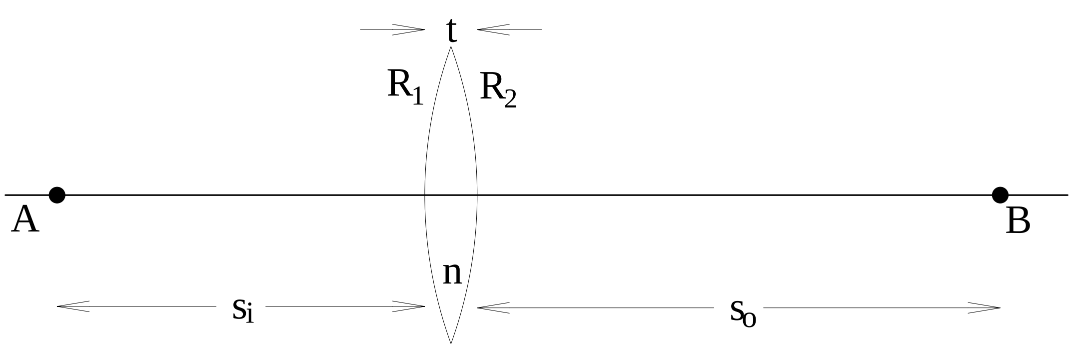


- Point A = object point
  Point B = image point
  Point V = vertex

- For geometry shown, s_o, s_i, R all positive

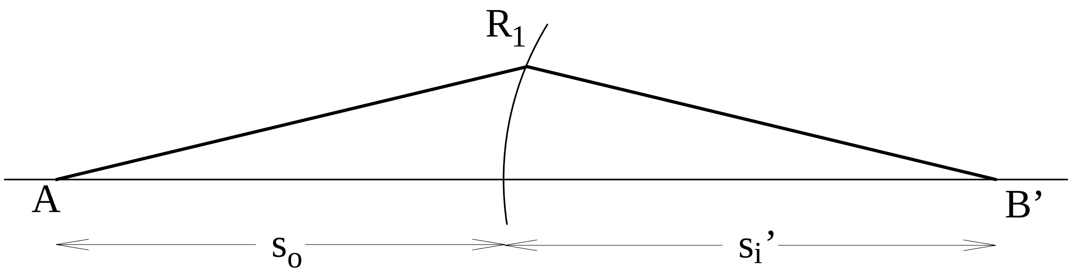
Thin Lenses (Hecht 5.2.3)

Don't usually want image in medium:
need two surfaces



Simplest case:
thickness of lens $t \ll R_1, R_2, s_o, s_i$
Neglect

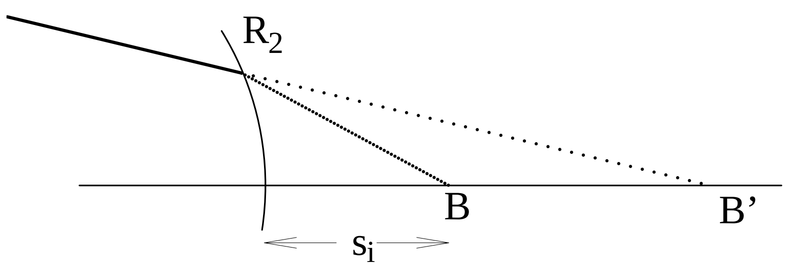
Solve one surface at a time



Assume incident medium = air

Then $\dfrac{1}{s_o} + \dfrac{n}{s_i'} = \dfrac{(n-1)}{R_1}$

Second surface:



Object of second surface = image from first
    = B' : to right of lens

OK: convention says $s_o' = -s_i'$
    called "virtual object"

(Also note: as drawn, $R_2 < 0$)

So, have

$$\frac{n}{s_o'} + \frac{1}{s_i} = \frac{1-n}{R_2}$$

$$-\frac{n}{s_i'} + \frac{1}{s_i} = \frac{1-n}{R_2}$$

$$-\left(\frac{n-1}{R_1} - \frac{1}{s_o}\right) + \frac{1}{s_i} = \frac{1-n}{R_2}$$

Gives

$$\frac{1}{s_o} + \frac{1}{s_i} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Define $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

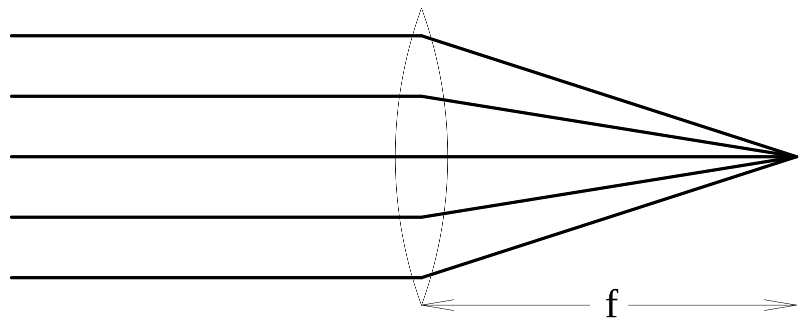
f = *focal length*

Thin lens equation:

$$\boxed{\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}}$$

Question: Where in this derivation did we use the assumption that the lens is thin?

In picture: f is image distance produced by collimated input



or object distance required to make collimated rays

Focal point = where collimated rays focused (on either side)

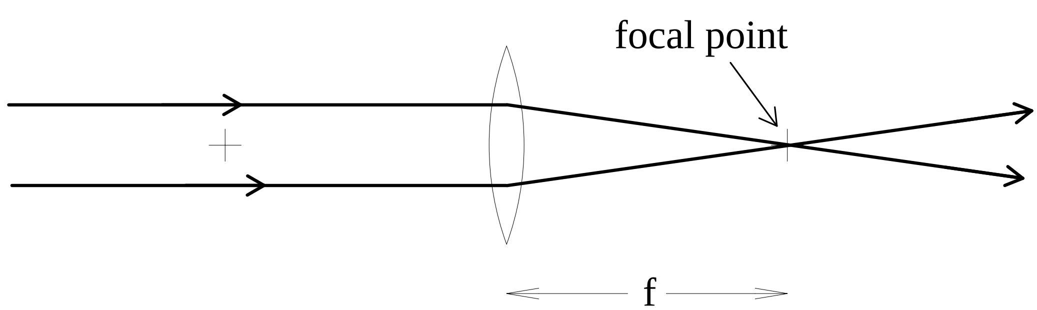
Lens usually specified by f

Thin Lens Behavior

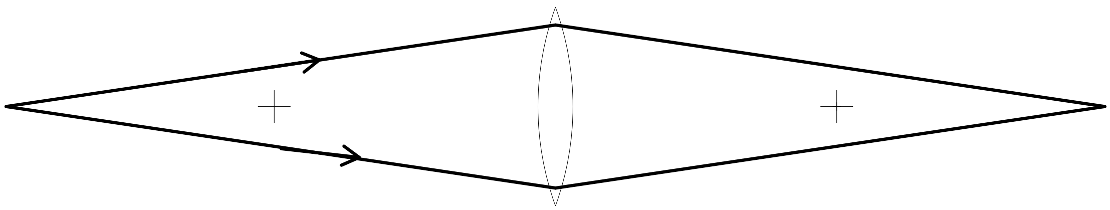
Thin lens equation valid for s_o, s_i, f either positive or negative

- Illustrate some cases

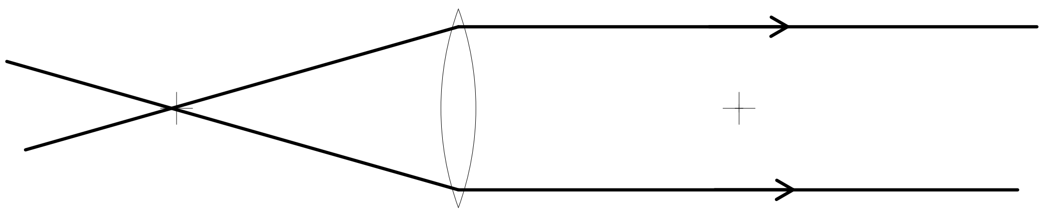
$f > 0$, $s_o = \infty$: then $s_i = f$



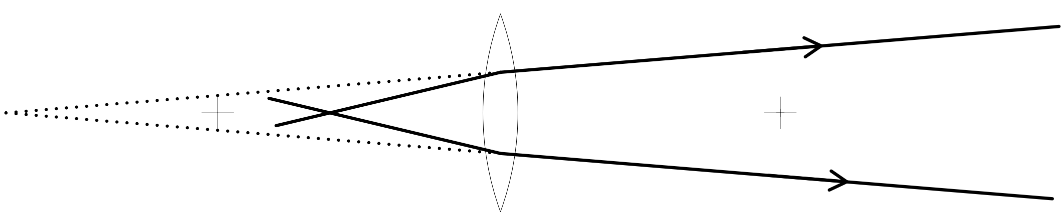
$f > 0$, $s_o > f$: then $s_i > f$



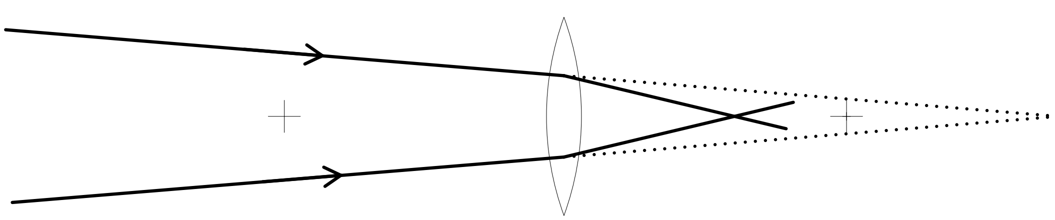
$f > 0$, $s_o = f$: then $s_i = \infty$



$f > 0$, $s_o < f$: then $s_i < 0$ "virtual image"

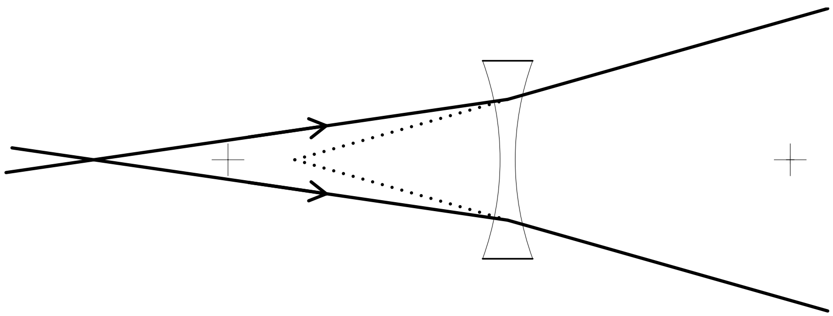


$f > 0$, $s_o < 0$: then $s_i < f$ "virtual object"

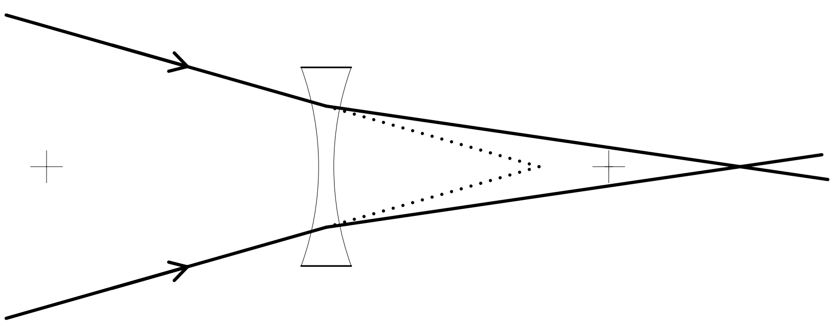


If $f < 0$, at least one of s_o, s_i is negative

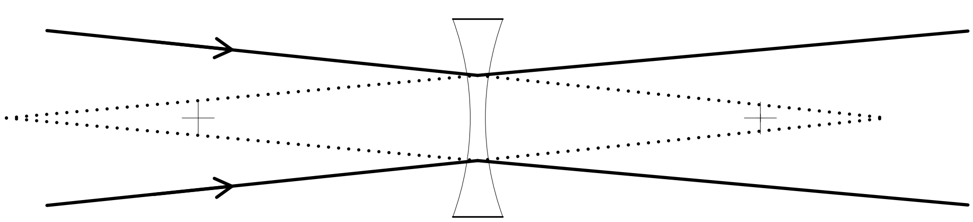
$f < 0, s_o > 0$: then $f < s_i < 0$



$f < s_o < 0$: then $s_i > 0$



$s_o < f < 0$: then $s_i < 0$



Demo!

Question: What happens if we put a lens right where the input rays are focused?

Can see that signs are tricky
- Real-life rays not left to right
- Gets worse with mirrors!

How I keep track:
light travels from upstream to downstream

- Object real if it is upstream of lens: $s_o > 0$
- Object virtual if it is downstream: $s_o < 0$

- Image real if it is downstream: $s_i > 0$
- Image virtual if it is upstream: $s_i < 0$

Summary:

- Ray = normal to wave front
- Ray optics: particles follow rays
- Ray optics accurate for large objects, short distances
- Fermat's principle gives ideal lenses
- Spherical lenses work in paraxial approximation
- Thin lens equation and sign convention important