Phys 531 Lecture 9 30 September 2004

Ray Optics II

Last time, developed idea of ray optics
approximation to wave theory

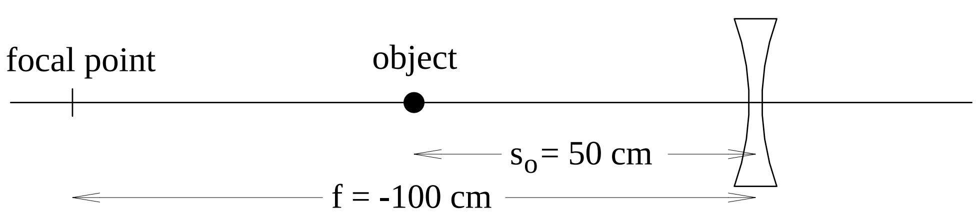
Introduced paraxial approximation:
rays with $\theta \ll 1$
Will continue to use

Started disussing imaging and lenses:

Thin lens equation $\dfrac{1}{s_o} + \dfrac{1}{s_i} = \dfrac{1}{f}$

Basic equation of paraxial optics

Example: Suppose a thin lens of focal length $f = -100$ cm is placed 50 cm in front of a small light bulb. Where will the image of the bulb be located?



Solution:
Real object upstream of lens, so $s_o = +50$ cm.

Then $\dfrac{1}{s_i} = \dfrac{1}{f} - \dfrac{1}{s_o} = \dfrac{1}{(-100\text{ cm})} - \dfrac{1}{50\text{ cm}} = -0.03\text{ cm}^{-1}$

So $s_i = -33.3$ cm

Negative, so image located 33.3 cm before lens
= 16.7 cm from object

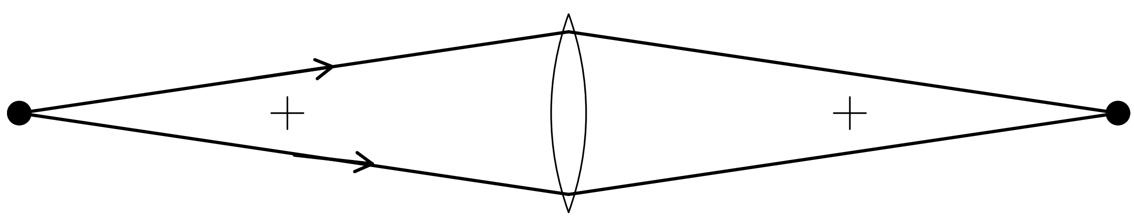
Today: continue with imaging

- Imaging extended objects
- Multiple lenses
- Mirrors
- Apertures

Next time:
Techniques for dealing with multi-lens systems

Finite Imaging (Hecht 5.2.3)

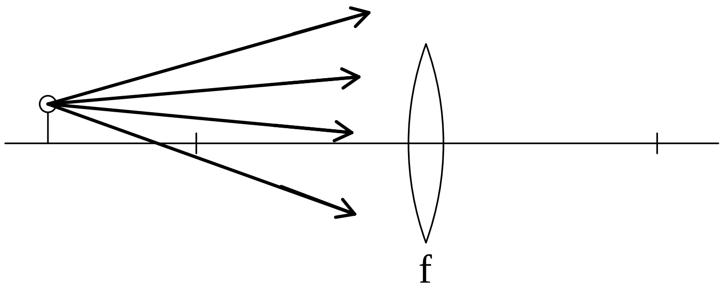
Before, all pictures showed point-to-point imaging



Points are on *optic axis*
= symmetry axis of lens

Finite object = collection of points
must deal with points that are off axis

Picture:



Construct image using *ray diagram*

Three simple rays:

- ray through lens center: undeviated
- ray through front focal point:
  becomes horizontal
- horizontal ray: hits back focal point

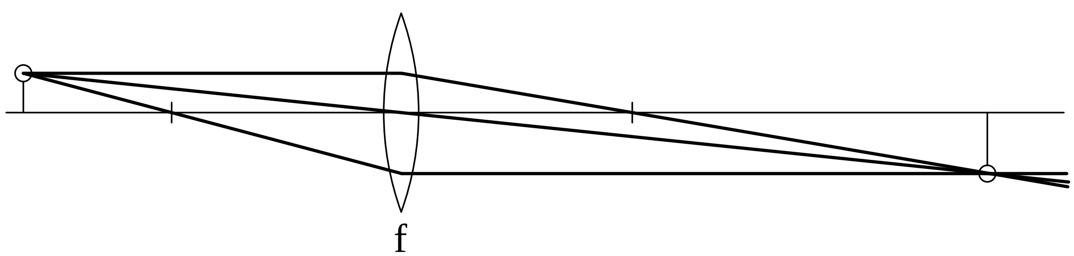


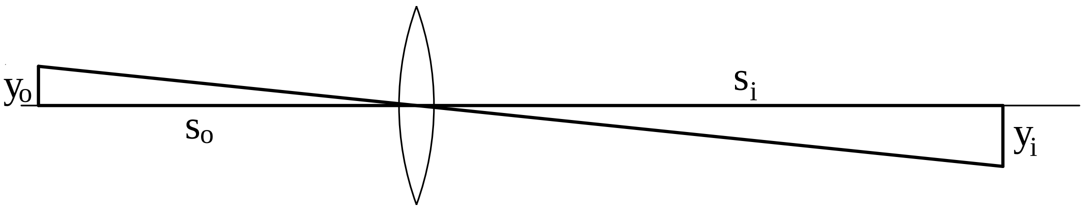
Image where rays intersect

Each point in object plane → point in image plane

Here, image inverted:
object height $y_o > 0$
maps to image height $y_i < 0$

Define *magnification* $m = \dfrac{y_i}{y_o}$

Get magnification from diagram:

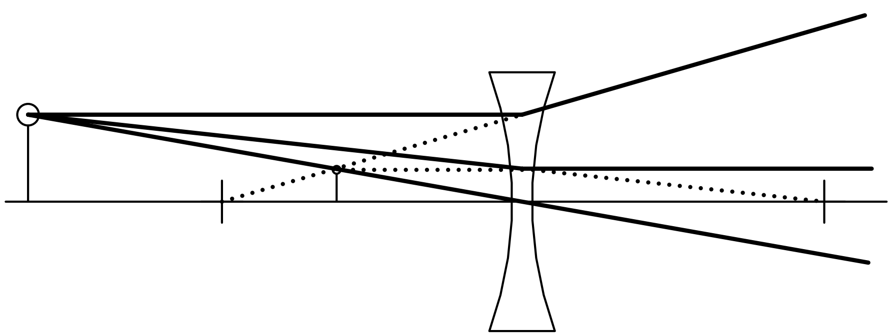


Triangles are similar, so $\dfrac{s_i}{y_i} = -\dfrac{s_o}{y_o}$

Then $m = \dfrac{y_i}{y_o} = -\dfrac{s_i}{s_o}$

with s_i determined by thin lens equation

For negative parameters, follow sign conventions:



Since $f < 0$, “front” and “back” focal points reversed

See that $s_i < 0$ and $m > 0$

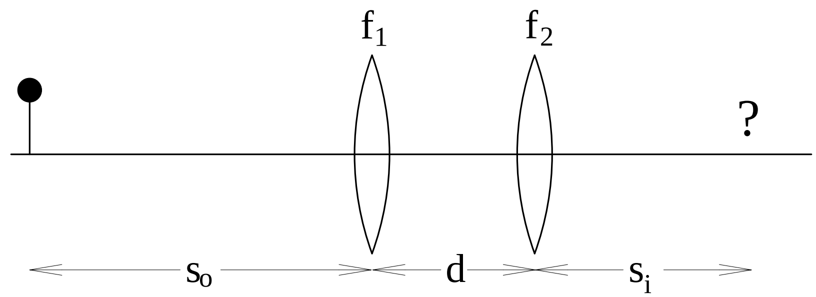
Ray diagrams good tool for understanding

Lens systems

If more than one lens:

apply thin lens law in succession

Consider two lenses f_1, f_2, separated by d



Find image distance s_i

First lens: image distance s_{i1}

$$\frac{1}{s_{i1}} = \frac{1}{f_1} - \frac{1}{s_o}$$

Second lens: object distance $s_{o2} = d - s_{i1}$

Then final image at s_i: $\frac{1}{s_i} = \frac{1}{f_2} - \frac{1}{s_{o2}}$

Combine, get

$$s_i = \frac{f_2 d s_o - f_1 f_2 d - f_1 f_2 s_o}{s_o d - s_o f_1 - s_o f_2 - d f_1 + f_1 f_2}$$

Not very enlightening

Simple case $d \to 0$:

$$s_{o2} = -s_{i1}$$

Then $\dfrac{1}{s_i} = \dfrac{1}{f_2} + \dfrac{1}{s_{i1}} = \dfrac{1}{f_2} + \left(\dfrac{1}{f_1} - \dfrac{1}{s_o}\right)$

So

$$\boxed{\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f_1} + \frac{1}{f_2}}$$

like single lens with $\dfrac{1}{f_{\text{tot}}} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

Define $\dfrac{1}{f}$ = *power of lens*

units: diopters ($\equiv$ m^{-1})

Lens powers add for adjacent lenses

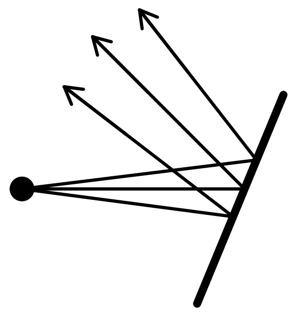
Question: My eyeglass prescription is -4 diopters. What is the focal length of my lenses?

Next time, see better tools to treat lens systems

For now, move on to other elements

Mirrors (Hecht 5.4)

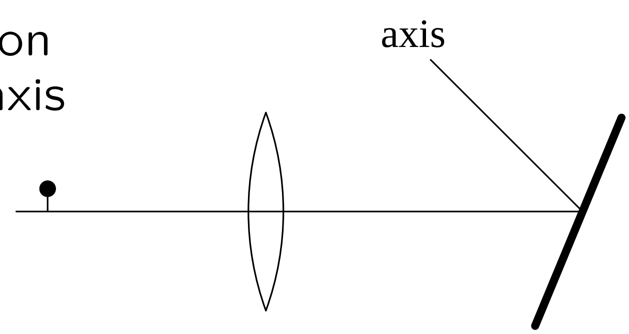
Flat mirrors just deflect rays



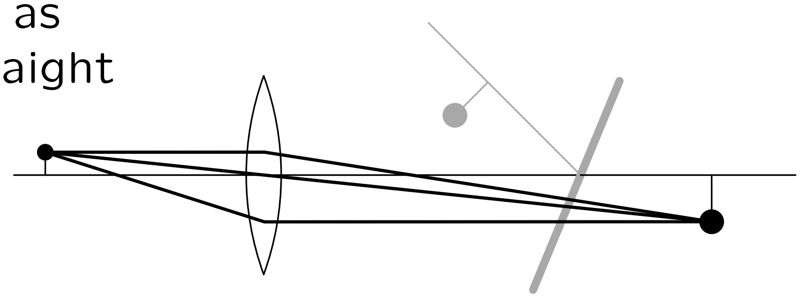
Makes ray tracing hard

Best approach:

(1) Use law of reflection
to find path of optic axis



(2) Trace rays as
if axis were straight

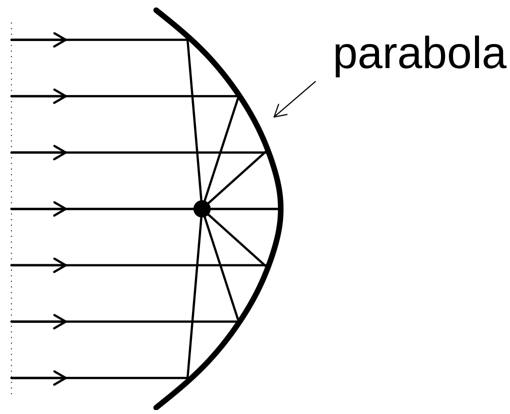


(3) Result correct relative to axis

Mirrors can be curved too

act like a lens

Get perfect imaging with aspheric mirror



Parabola = points equidistant from focus and line

so $\mathcal{S}$ = constant for all rays

Aspheric surfaces still expensive:

spherical mirrors more common

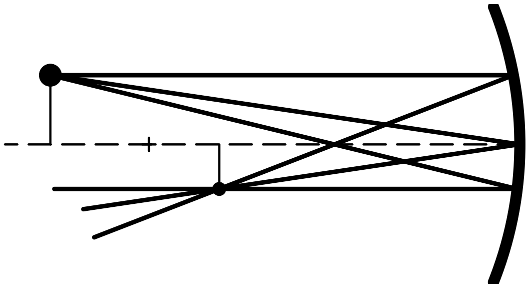
work for paraxial rays

Analyze with law of reflection or Fermat's principle

Result:

$$\frac{1}{s_o} + \frac{1}{s_i} = -\frac{2}{R}$$

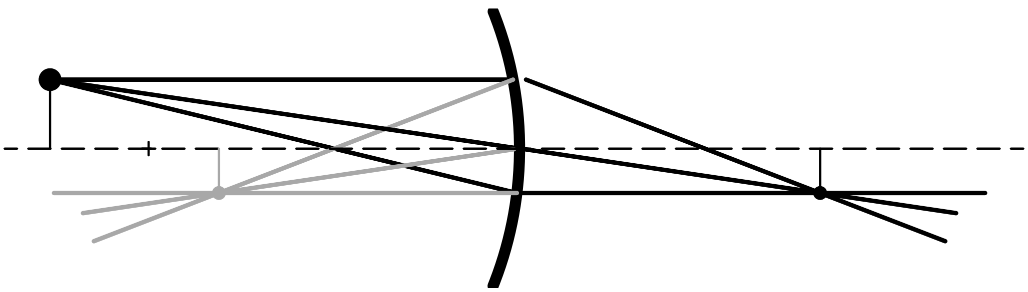
Just like thin lens: $f \to -R/2$



Here $R < 0$: like $f > 0$, get real image

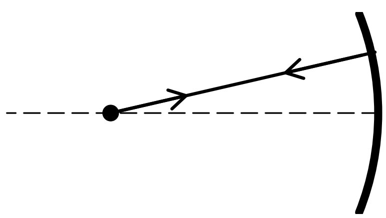
But optic axis also folded

Lens equivalent:



Trick to remember factor of 2:

if object at center of curvature, so is image

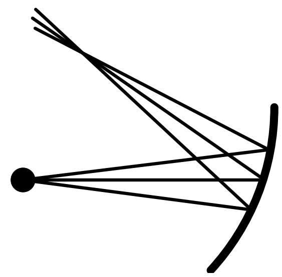


Then $s_o = s_i = -R$ (since $R < 0$ here)

and

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} = -\frac{2}{R}$$

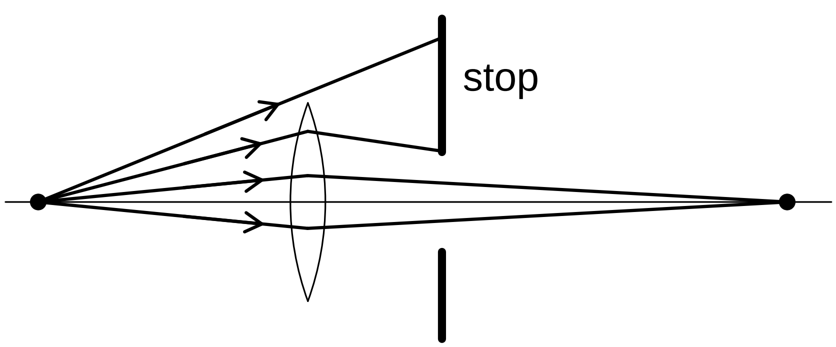
Question: Could you tilt a curved mirror and use it to both fold axis and focus rays?



Stops

stop = aperture = hole

Formally:
stop = object that limits rays reaching image



Lens edge always a stop
often introduce additional ones

Stops important for two questions:

- How bright will image be?
  = how much light is collected
- What is field of view?
  = what area of object is imaged

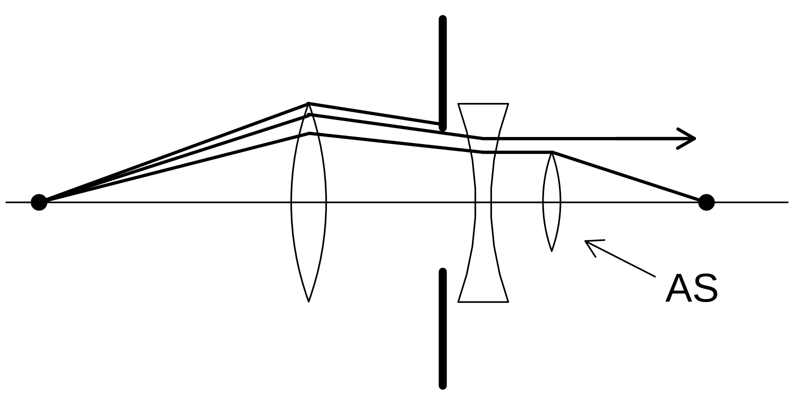
Both important for real system design

Assume points close to optic axis
off-axis points: vignetting (Hecht pgs. 172-3)

Image Brightness:

Generally many stops in system

Define *aperture stop* ($\equiv$ AS)
= the one limiting stop



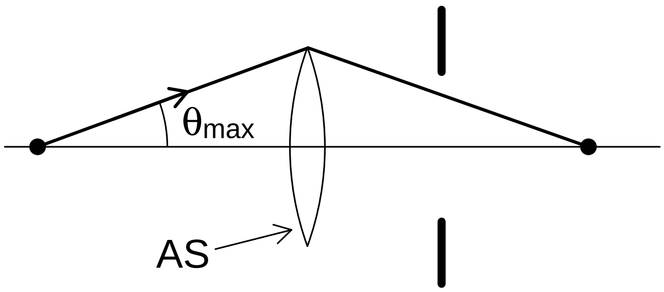
In complicated system, trace rays to determine

Aperture stop sets amount of light collected

Suppose AS = first element

then only rays hitting element are imaged

Defines acceptance angle θ_{max}



Characterize source by *brightness* B

$$B = \frac{\text{power}}{\text{(source area)(solid angle)}}$$

units W/(m^2 sr)

Question: What's a steradian (sr)?

Typical light bulb:

P = 100 W

surface area = $4\pi\times$(3 cm)^2

radiates into 4π sr

So $B \approx 700$ W/(m^2 sr)

System accepts solid angle $\pi\theta^2_{\text{max}}$

Power into system = $B\times$(source area)$\times\pi\theta^2_{\text{max}}$

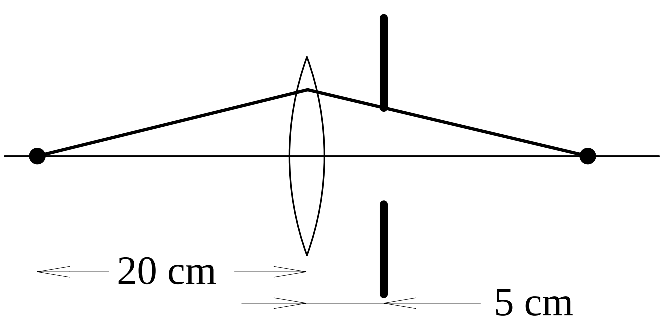
Gives image intensity

What if first element $\neq$ AS?

Define *entrance pupil*
= image of AS from object

System acts like first element is entrance pupil

Example:



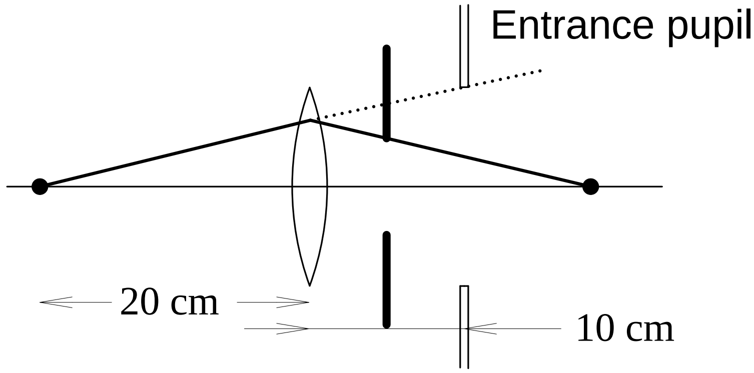
Say $f = 10$ cm, stop radius = 0.5 cm

Find image of stop through lens:
$s_o = 5$ cm $\Rightarrow s_i = -10$ cm

Magnification $m = -s_i/s_o = 2$

So entrance pupil located 10 cm behind lens

radius = 1 cm



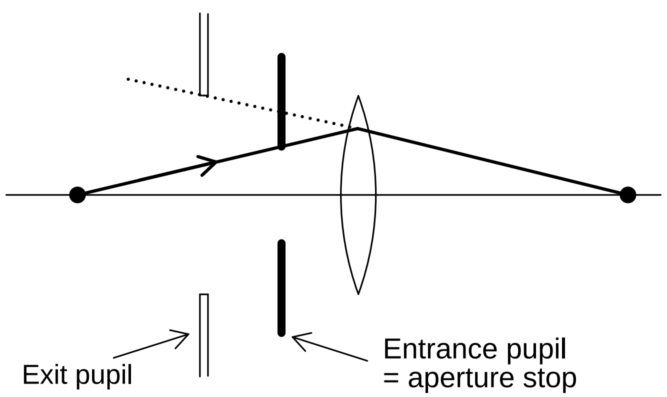
Defines rays accepted by system

here $\theta_{\text{max}} = \tan^{-1}(1/30) \approx 0.033$ rad

solid angle $= \pi\theta^2_{\text{max}} = 0.0034$ sr

Similarly, define *exit pupil*

= image of AS seen from image of system



Exit pupil acts like “window”

all rays from object pass through window

Exit pupil important for systems viewed by eye
microscope, telescope, binoculars, etc.

Exit pupil like window

- if pupil small, far away:
  observe small disk surrounded by darkness

See in cheap binoculars

- if pupil large, close to eye:
  many rays don't enter eye ... wasted money

Ideal system: exit pupil aligned to pupil of eye

- 3-5 mm diameter
- located about 1 cm behind eyepiece

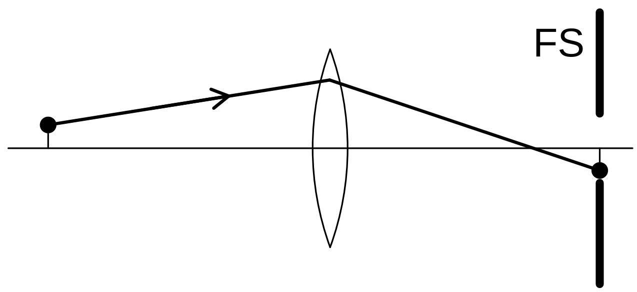
(Eyepiece = last element before eye
Distance eyepiece to exit pupil = eye relief)

Good binoculars:
- image fills view
- comfortable to look through

Generally: when combining two systems, make exit pupil of first = entrance pupil of second

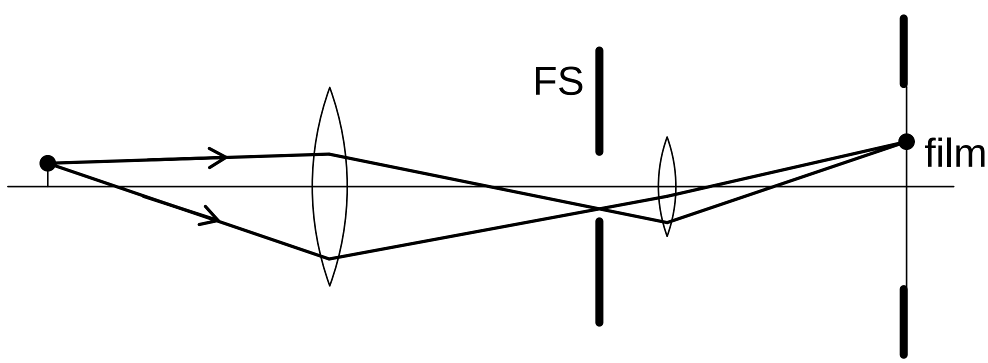
Still have second question:
what is field of view?

Define *field stop* (FS) =
stop that limits which object points are imaged



Often, field stop = edge of detector
(film, CCD sensor, retina of eye, etc.)

But not always:



Good field stop always in an image plane
(final or intermediate)

Field stops useful for non-imaging detectors

- photodiodes, PMT, bolometers

Use field stop to eliminate background light

Question: Photographers use a stop to set the exposure level of a camera. Does this refer to the field stop or the aperture stop?

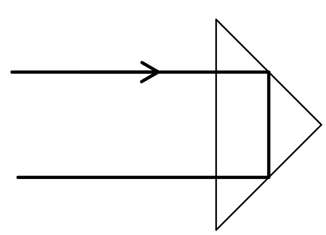
Also, both AS and FS impact aberrations
(imaging errors)

Idea: use stops to block non-paraxial rays

Last element: prisms
comment briefly

Two uses: reflection and dispersion

Reflecting: either TIR or mirror coatings
nice way to hold mirrors close together

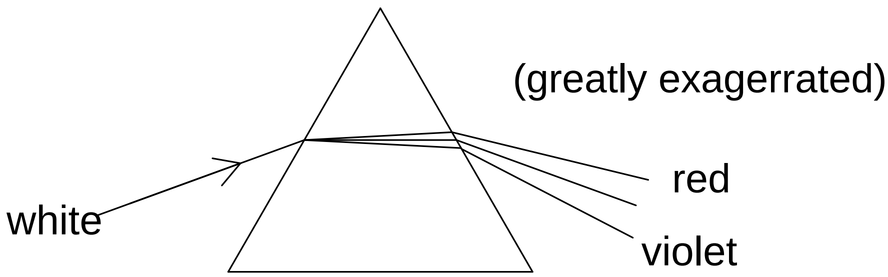


See Hecht 5.5.2 for more

Dispersing prisms: familiar rainbow effect

Use index $n = n(\omega)$

Snell's law $\Rightarrow \theta = \theta(\omega)$



Analysis straighforward (Hecht 5.5.1)

More often use diffraction gratings

Summary:

- Ray diagrams help analyze off-axis imaging
- Analyze multiple lenses in sequence
- Curved mirrors act much like lenses
- Stop = limiting edge
- Aperture stop sets brightness of image
- Field stop sets field of view