

$$1. \quad r_{\perp} = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t}$$

$$r_{\parallel} = \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_t \cos \theta_i + n_i \cos \theta_t}$$

$$n_i = 1.7 \quad n_t = 1.0$$

$$\Theta_c = \sin^{-1} \frac{n_i}{n_t} = 36^\circ$$

$$\text{So } \theta_i = 46^\circ \quad \cos \theta_i = 0.695$$

$$\begin{aligned} \theta_t &= \sin^{-1} \left[ \frac{n_i}{n_t} \sin \theta_i \right] = \sin^{-1}(1.22) \\ &= \frac{\pi}{2} - 0.656; \end{aligned}$$

$$\cos \theta_t = 0.704;$$

$$\begin{aligned} \text{So } r_{\perp} &= \frac{1.7 \times 0.695 - 0.704i}{1.7 \times 0.695 + 0.704i} = \frac{1.182 - 0.704i}{1.182 + 0.704i} \\ &= 0.476 - i0.879 \\ &= 1.0L - 61.6^\circ \end{aligned}$$

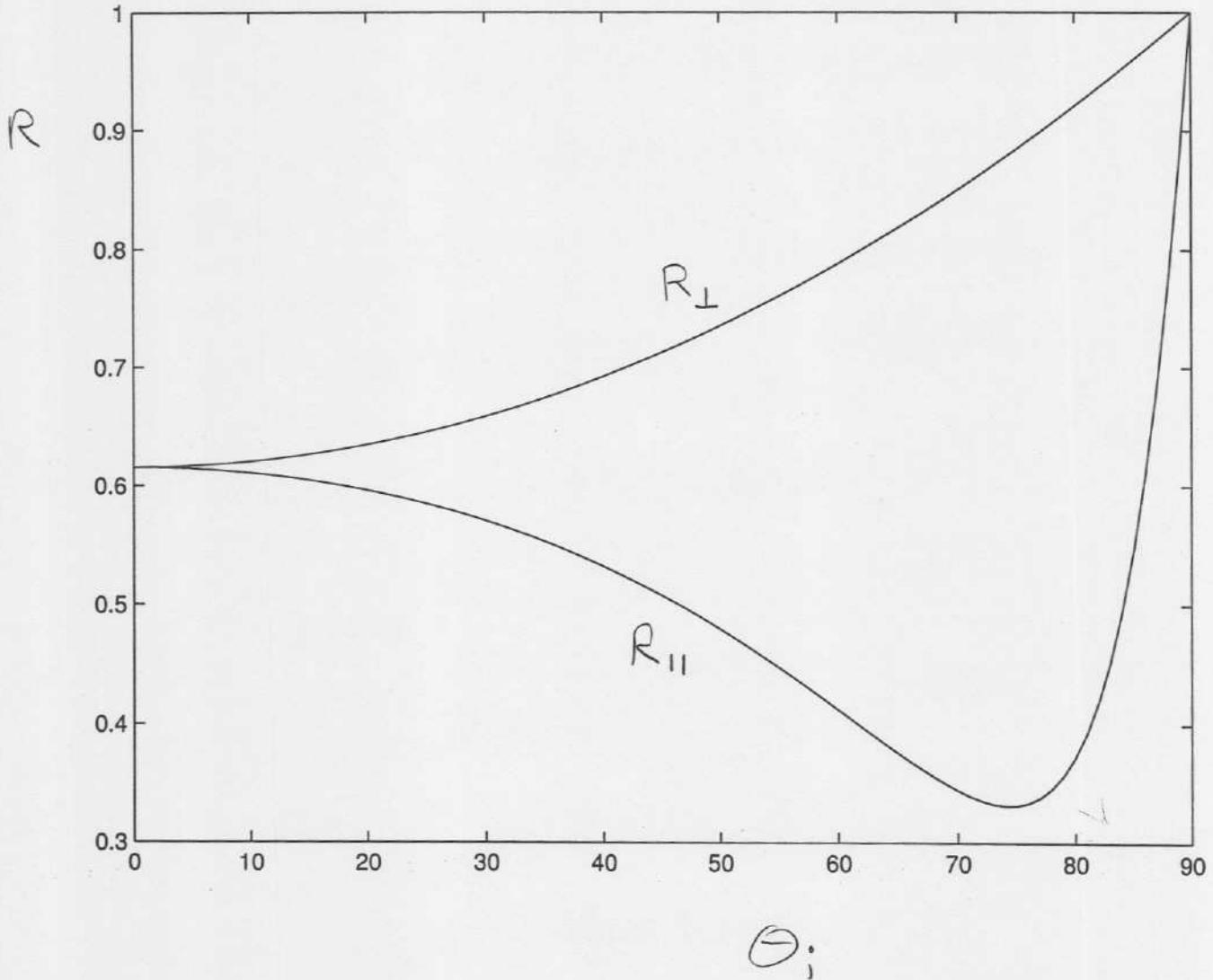
$$\begin{aligned} r_{\parallel} &= \frac{0.695 - 1.7 \times 0.704i}{0.695 + 1.7 \times 0.704i} = \frac{0.695 - 1.197i}{0.695 + 1.197i} \\ &= 1.0L - 119.7^\circ \end{aligned}$$

So phase difference is

$$\phi_{\perp} - \phi_{\parallel} = -61.6^\circ + 119.7^\circ = \boxed{58.1^\circ}$$

2. Use Matlab

```
n = 1.8 + 3.3i;  
qi = [0:90]*pi/180;  
qt = asin(sin(qi)/n);  
  
ci = cos(qi);  
ct = cos(qt);  
  
rperp = (ci - n*ct)./(ci + n*ct);  
rpara = (n*ci - ct)./(n*ci + ct);  
  
plot(qi*180/pi,abs(rperp).^2,qi*180/pi,abs(rpara).^2);
```



3. PCX: (3)

Oriented



$$R_1 = \infty \quad R_2 = -R$$

$$\frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{n-1}{R}$$

$$f = \frac{R}{n-1}$$

Oriented



$$R_1 = R \quad R_2 = \infty$$

$$\frac{1}{f} = (n-1) \left( \frac{1}{R} - \frac{1}{\infty} \right) = \frac{n-1}{R}$$

no difference

BCX:



$$R_1 = R \quad R_2 = -R$$

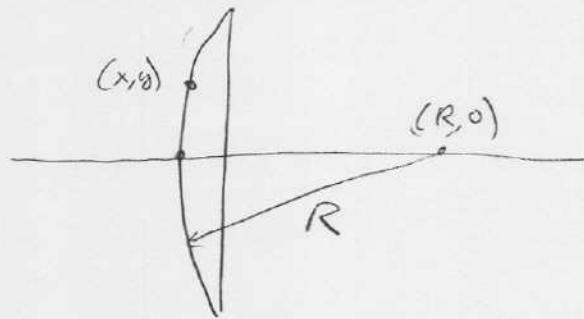
$$\frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{2(n-1)}{R}$$

$$f = \frac{R}{2(n-1)}$$

4.

(4)

a)



Need to know  $x$  as function of  $y$ .

$$\text{Sphere: } (x-R)^2 + y^2 = R^2$$

$$x^2 - 2xR + y^2 = 0$$

Quadratic equation

$$x = \frac{1}{2} [2R \pm \sqrt{4R^2 - 4y^2}]$$

$$= R [1 \pm \sqrt{1 - \frac{y^2}{R^2}}]$$

$$\approx R [1 \pm (1 - \frac{y^2}{2R^2})]$$

Want  $x$  small, so "-" root

$$x \approx \frac{y^2}{2R}$$

$$\text{Then } t = t_0 - x = \boxed{t_0 - \frac{y^2}{2R}}$$

$$b) f = \frac{R}{n-1}$$

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

$$\text{so } \frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{1}{f} - \frac{1}{2f} = \frac{1}{2f}$$

$$\boxed{s_i = 2f}$$

c) Optical path length

$$S = \sqrt{(2f)^2 + y^2} + (t_0 - t) + nt + \sqrt{(2f)^2 + y^2}$$

↑                      ↑                      ↑                      ↑  
 to lens              air gap              glass              to image

$$S = 2\sqrt{4f^2 + y^2} + t_0 + (n-1)\left[t_0 - \frac{y^2}{2R}\right]$$

$$\begin{aligned} \text{use } \sqrt{4f^2 + y^2} &= 2f\sqrt{1 + \frac{y^2}{4f^2}} \approx 2f\left(1 + \frac{y^2}{8f^2}\right) \\ &= 2f + \frac{y^2}{4f} \end{aligned}$$

$$S = 4f + \frac{y^2}{2f} + nt_0 - (n-1)\frac{y^2}{2R}$$

$$\text{But } f = \frac{R}{n-1}, \text{ so}$$

$$\begin{aligned} S &= 4f + nt_0 + \frac{y^2}{2}\left(\frac{n-1}{R}\right) - (n-1)\frac{y^2}{2R} \\ &= \boxed{4f + nt_0} \end{aligned}$$

Independent of  $y$ , as required by Fermat

(6)

5. Find image from first lens:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{1}{10\text{cm}} - \frac{1}{15\text{cm}} = \frac{1}{30\text{cm}}$$

$$s_i = 30\text{ cm}$$

If lenses separated by 10 cm, then

$$\text{for } L_2, \quad s_o = 10\text{cm} - 30\text{cm} = -20\text{cm}$$

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{1}{-20\text{cm}} - \frac{1}{-20\text{cm}} = 0$$

$$s_o \quad s_i = \infty$$

Image plane at  $\infty$ : light is collimated

6.  $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$

$$s_o \quad \frac{1}{s_i} = \frac{1}{30\text{cm}} - \frac{1}{20\text{cm}} = \frac{1}{-60\text{cm}}$$

$$s_i = -60\text{cm} \quad \text{virtual}$$

$$\text{Magnification } m = -\frac{s_i}{s_o} = -\frac{(-60)}{20} = \boxed{3}$$

⑦

Ray Diagram  $f = 10\text{ cm}$ 