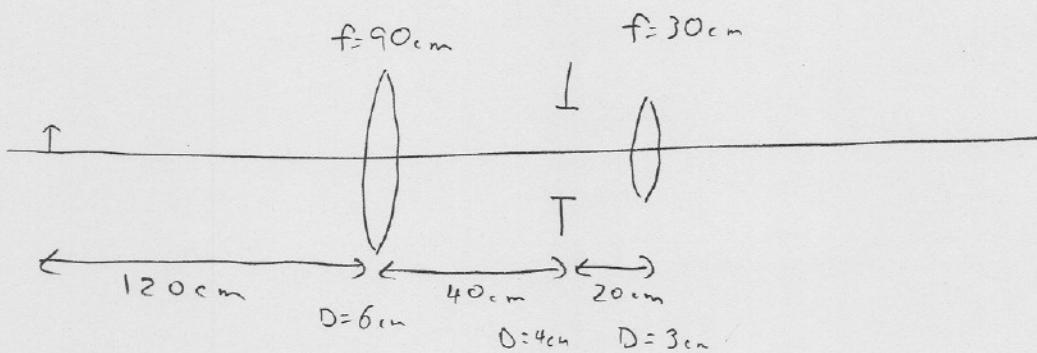
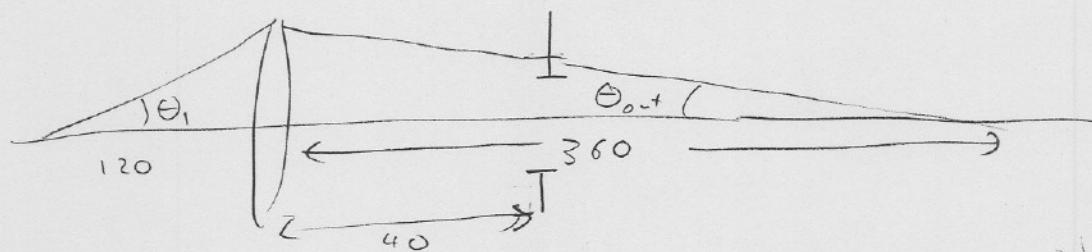


1.



- a) Three possibilities for AS:  $L_1$ ,  $L_2$ , or diaphragm  
 Can find either using ray tracing or  
 by considering pupils.  
 I'll show both ways.

Ray tracingTry ray hitting rim of  $L_1$ 

$$\Theta_i = \frac{3}{120} = 2.5 \text{ mrad}$$

$$\text{Image at } \frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{1}{90} - \frac{1}{120} = \frac{1}{360 \text{ cm}}$$

$$\text{So } \Theta_{o+} = \frac{3}{360} = 8.3 \text{ mrad}$$

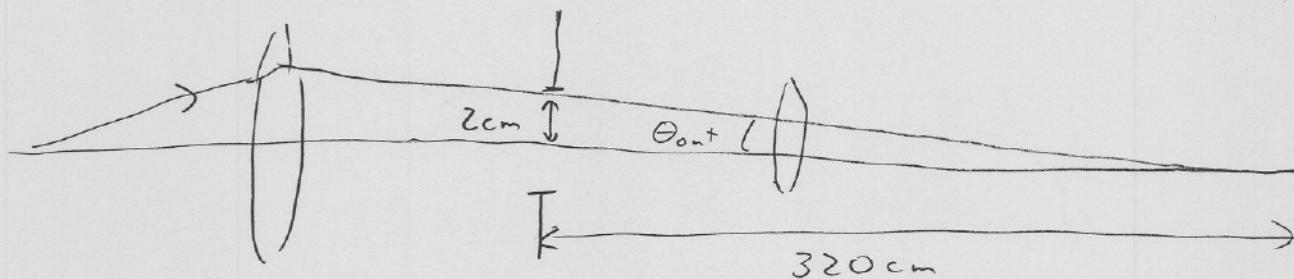
$$\text{At diaphragm, ray has height} = \frac{320}{360} \times 3 = 2.66 \text{ cm}$$

$$\text{But hole radius} = 2 \text{ cm}$$

(2)

So ray hits diaphragm:  $L_1$  is not AS

Then consider ray hitting edge of diaphragm



$$\text{Then } \theta_{\text{out}} = \frac{2}{320} = 6.3 \text{ mrad}$$

$$\text{At } L_2, y = \frac{300}{320} \times 2 = 1.875 \text{ cm}$$

Radius of  $L_2 = 2 \text{ cm}$ , ray pass through  $L_2$

Conclude that diaphragm is AS

### Entrance Pupil method

Pupil for  $L_1$  is rim of lens

angle  $\Theta = 25 \text{ mrad}$  as above

Pupil for diaphragm:

Image diaphragm through  $L_1$ :

$$S_o = 40 \text{ cm} \quad f = 90 \text{ cm}$$

$$\frac{1}{S_i} = \frac{1}{90} - \frac{1}{40} = -\frac{1}{72 \text{ cm}}$$

(3)

So image is 72 cm to right of L,  
 = 192 cm from object

$$\text{magnification: } -\frac{s_i}{s_o} = \frac{72}{40} = 1.8$$

$$\text{Image radius} = 1.8 \times 2 \text{ cm} = 3.6 \text{ cm}$$

$$\text{angle } \Theta: \frac{3.6 \text{ cm}}{192 \text{ cm}} = \boxed{18.8 \text{ mrad}}$$

Pupil for L<sub>2</sub>: image through L<sub>1</sub>, so

$$s_o = 60 \text{ cm} \quad f = 90 \text{ cm}$$

$$\frac{1}{s_i} = \frac{1}{90} - \frac{1}{60} = -\frac{1}{180}$$

Image is 300 cm from object

$$m = -\frac{s_i}{s_o} = \frac{180}{60} = 3$$

$$\text{Radius} = 3 \times 2 = 6 \text{ cm}$$

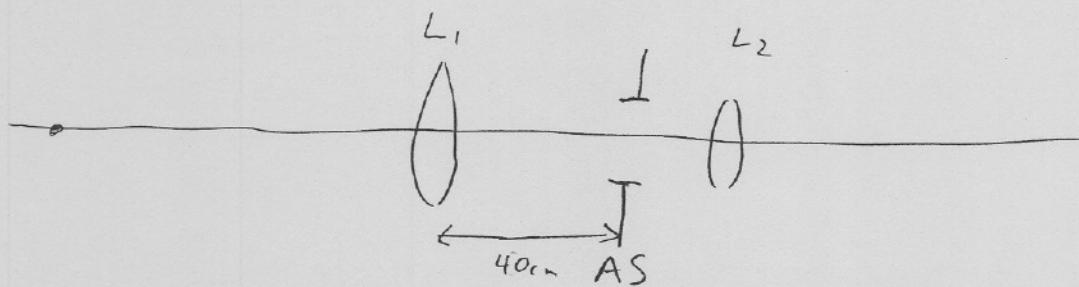
$$\text{angle} = \frac{6 \text{ cm}}{300 \text{ cm}} = \boxed{20 \text{ mrad}}$$

AS is element with smallest pupil

= diaphragm

3½

b) Work through again:



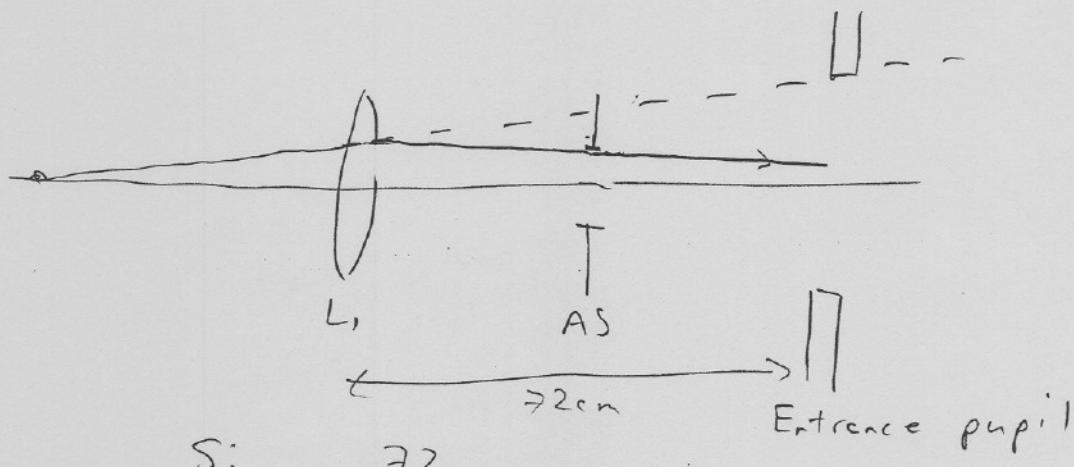
Entrance pupil is image of AS produced by  $L_1$ , going backwards through system.

$$\text{So here } S_o = +40\text{cm}$$

$$\frac{1}{S_i} = \frac{1}{f} - \frac{1}{S_o} = \frac{1}{90\text{cm}} - \frac{1}{40\text{cm}} = -\frac{1}{72\text{cm}}$$

Since  $S_i < 0$ , image is "upstream" of lens

But here we're going from right to left,  
so image is to the right



$$m = -\frac{S_i}{S_o} = \frac{72}{40} = 1.8$$

So entrance pupil is  $1.8 \times 4 = \boxed{7.2 \text{ cm diameter}}$

Located  $\boxed{192\text{cm from object}}$

3  $\frac{3}{4}$

c) Exit pupil: image AS through  $L_2$

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{1}{30} - \frac{1}{20} = -\frac{1}{60\text{cm}}$$

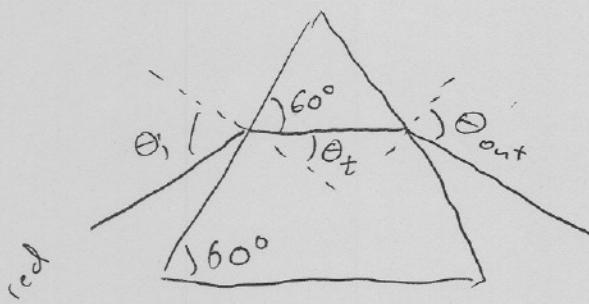
So exit pupil is 60 cm to left of  $L_2$

$$m = \frac{60}{20} = 3$$

exit pupil diameter = 12 cm

(4)

2. Equilateral triangle has angles =  $60^\circ$



So for red light,

$$\theta_t = 30^\circ$$

$$\begin{aligned} \text{So } \sin \theta_i &= 1.51 \sin 30^\circ \\ &= 0.755 \end{aligned}$$

$$\theta_i = 49.0^\circ$$

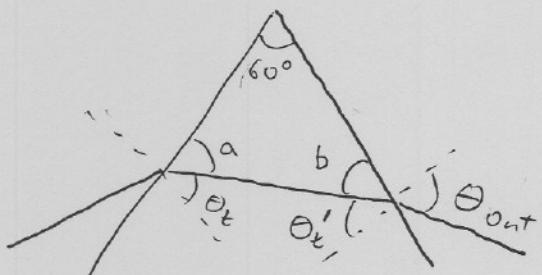
By symmetry,  $\theta_{\text{out}} = \theta_i = 49.0^\circ$

For violet light:

$$\sin \theta_t = \frac{1}{1.53} \sin 49^\circ$$

$$\theta_t = 29.57^\circ$$

Need  $\theta_t'$  = angle at output surface



From picture

$$a = 90^\circ - \theta_t$$

$$a + b + 60^\circ = 180^\circ$$

$$\begin{aligned} b &= 120^\circ - a = 120^\circ - 90^\circ + a \\ &= 30^\circ + a \end{aligned}$$

$$\begin{aligned} \theta_t' &= 90^\circ - b = 60^\circ - a \\ &= 30.43^\circ \end{aligned}$$

$$\text{So } \sin \theta_{\text{out}} = 1.53 \sin 30.43^\circ$$

$$\theta_{\text{out}} = 50.80^\circ$$

$$\Delta \theta = \theta_{\text{violet}} - \theta_{\text{red}} = \boxed{1.08^\circ}$$

(5)

3. Calculate ray matrix:



$$M_v = \begin{bmatrix} 1 & 0 \\ -\frac{R}{n} & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{1-n}{R} \\ 0 & 1 \end{bmatrix}$$

Note back surface has no effect, and  $R>0$  here.

$$M_v = \begin{bmatrix} 1 & \frac{1-n}{R} \\ \frac{R}{n} & 1 + \frac{1-n}{n} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{Focal length } f = -\frac{1}{b} = \frac{R}{n-1}$$

$$\text{Front focal length} = -\frac{a}{b} = \boxed{\frac{R}{n-1}}$$

$$\text{Back focal length} = -\frac{d}{b} = \frac{R}{n-1} \left( 1 + \frac{1-n}{n} \right)$$

$$= R \left( \frac{1}{n-1} - \frac{1}{n} \right)$$

$$= R \left( \frac{n - (n-1)}{n(n-1)} \right)$$

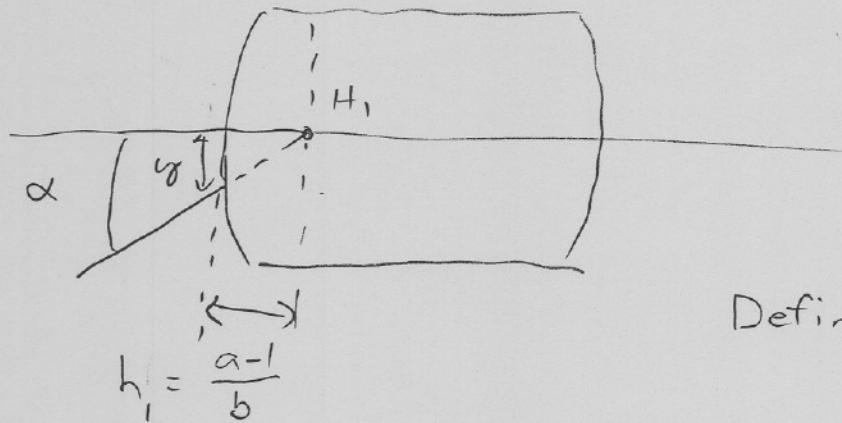
$$\boxed{bf_l = \frac{R}{n(n-1)}}$$

(6)

$$4. \text{ Front focal length} = -\frac{a}{b}$$

$$\text{Focal length} = -\frac{1}{b}$$

So principal point located distance f-ffl  
behind front vertex



$$\text{Define } h_1 = \frac{a-1}{b}$$

So incident ray with angle  $\alpha$  aimed  
at  $H_1$  is

$$\begin{bmatrix} \alpha \\ -\alpha h_1 \end{bmatrix}$$

$$\text{Output ray} = M_U \begin{bmatrix} \alpha \\ -\alpha h_1 \end{bmatrix} = \alpha \begin{bmatrix} a-h_1, b \\ c-h_1 d \end{bmatrix} = \begin{bmatrix} \alpha_{out} \\ \beta_{out} \end{bmatrix}$$

$$\text{Note } a+h_1, b = a - \frac{(a-1)}{b} b = 1$$

$$\text{So } \boxed{\alpha_{out} = \alpha_{in}}$$

(7)

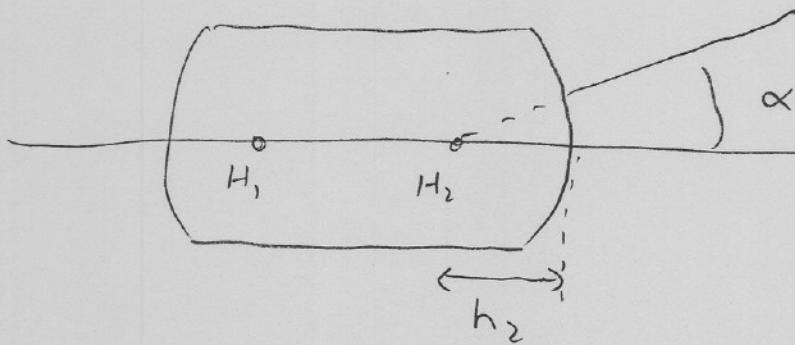
$$\begin{aligned}
 y_{\text{out}} &= \alpha(c - h_1 d) \\
 &= \alpha\left(c - \frac{a-1}{b}d\right) = \alpha\left(\frac{bc - ad + d}{b}\right) \\
 &= \alpha\left(\frac{d-1}{b}\right)
 \end{aligned}$$

since  $ad - bc = 1$

But, distance from  $H_2$  to back vertex

$$\begin{aligned}
 &= f - b f l = -\frac{1}{b} - \left(-\frac{d}{b}\right) = \frac{d-1}{b} \\
 &= h_2
 \end{aligned}$$

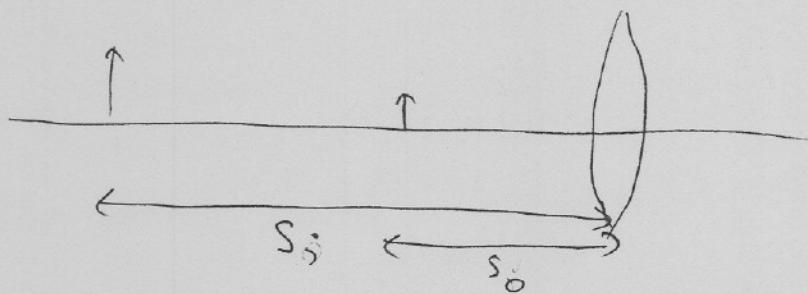
So  $y_{\text{out}} = \alpha h_2$



Ray appears to come from back principle point

(8)

5.



Want  $s_i = -125 \text{ cm}$  (virtual image)

Have  $s_o = 25 \text{ cm}$

$$\begin{aligned} s_o & \quad \frac{1}{f} = \frac{1}{25} - \frac{1}{125} = \frac{1}{31.25 \text{ cm}} \\ & \qquad \approx 3.2 \text{ m}^{-1} \\ & \qquad = \boxed{3.2 \text{ diopters}} \end{aligned}$$

6. a) Use  $\mathcal{R}_{n_2 \rightarrow 1} = \begin{bmatrix} 1 & \frac{n_1 - n_2}{R} \\ 0 & 1 \end{bmatrix}$        $\mathcal{T}_1 = \begin{bmatrix} 1 & 0 \\ \frac{d_1}{n} & 1 \end{bmatrix}$

$$R_1 = 34.43 \text{ mm}$$

$$d_1 = 1.5 \text{ mm}$$

$$R_2 = 22.22 \text{ mm}$$

$$d_2 = 2.8 \text{ mm}$$

$$R_3 = -30.62 \text{ mm}$$

$$d_3 = 1.5 \text{ mm}$$

$$\begin{aligned} M &= \mathcal{R}_{n_2 \rightarrow 1}(-R_1) \mathcal{T}_1\left(\frac{d_1}{n_2}\right) \mathcal{R}_{n_1 \rightarrow n_2}(-R_2) \mathcal{T}_1\left(\frac{d_2}{n_1}\right) \mathcal{R}_{n_2 \rightarrow n_1}(-R_3) \\ &\times \mathcal{T}_1(d_3) \mathcal{R}_{n_1 \rightarrow 1}(R_3) \mathcal{T}_1\left(\frac{d_2}{n_1}\right) \mathcal{R}_{n_2 \rightarrow n_1}(R_2) \mathcal{T}_1\left(\frac{d_1}{n_2}\right) \mathcal{R}_{1 \rightarrow n_2}(R_1) \end{aligned}$$

Using Matlab, get

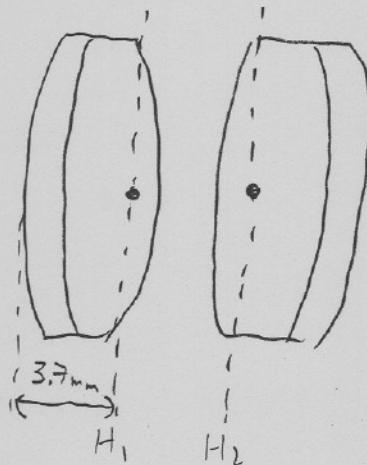
$$M = \begin{bmatrix} 0.7993 & -0.0542 \text{ mm}^{-1} \\ 6.6597 \text{ mm} & 0.7993 \end{bmatrix}$$

(check,  $\det M = 1 \checkmark$ )

b)  $f = -\frac{1}{B} = 18.45 \text{ mm}$

$$ffl = -\frac{A}{B} = 14.75 \text{ mm} = bfl$$

So the principle planes are  $18.45 - 14.75 = 3.7 \text{ mm}$   
from vertices



(9)

```

r1 = 34.43;
r2 = 22.22;
r3 = -30.62;
d1 = 1.5;
d2 = 2.8;
d3 = 1.5;
n1 = 1.5189;
n2 = 1.6771;
R1 = [1 (1-n2)/r1; 0 1];
R2 = [1 (n2-n1)/r2; 0 1];
R3 = [1 (n1-1)/r3; 0 1];
R4 = [1 -(1-n1)/r3; 0 1];
R5 = [1 -(n1-n2)/r2; 0 1];
R6 = [1 -(n2-1)/r1; 0 1];
t1 = [1 0; d1/n2 1];
t2 = [1 0; d2/n1 1];
t3 = [1 0; d3 1];
m = R6*t1*R5*t2*R4*t3*R3*t2*R2*t1*R1

```

c) For eyepieces,  $MP = \frac{254\text{mm}}{f} = \frac{254}{18.45} = \boxed{13.8 \times}$

d) Image AS through eyepiece

$$S_o = 176\text{ mm} + 18.45\text{ mm} = 194.45\text{ mm}$$

↑  
 $f$ : distance from  
 field stop to  
 front principle plane

$$\frac{1}{S_i} = \frac{1}{f} - \frac{1}{S_o} = \frac{1}{18.45} + \frac{1}{194.45} = \frac{1}{20.38\text{ mm}}$$

So exit pupil is 20.38 mm from back principle plane

$$\begin{aligned} \text{Distance from back vertex} &= 20.38\text{ mm} - 3.7\text{ mm} \\ &= \boxed{16.7\text{ mm}} \\ &= \text{eye relief} \end{aligned}$$

$$\begin{aligned} \text{Size of exit pupil} &= 1\text{ cm} \times \frac{S_i}{S_o} = 1\text{ cm} \times \frac{20.38}{194.5} \\ &= \boxed{1.0\text{ mm}} \end{aligned}$$