Phys 532 Midterm Exam

1. Beam is too big : $w_0 \approx \frac{\lambda}{\Theta}$ for focussing angle Θ

so increase $\Theta \approx \frac{w_{in}}{f}$ for incident waist w_{in}

So, decrease f = $\boxed{\text{shorter focal length}}$

2. $\Delta\nu_L = \frac{c}{nL}$ L = round-trip length = 10 cm

n = 1.5

$= \frac{3\times10^{10}\text{ cm/s}}{15\text{ cm}} = \boxed{2\text{ GHz}}$

3. From HW 3, (or Yariv 8.2-7) $\alpha = \frac{\chi''\omega}{nc}$ $\chi = \chi' - i\chi''$

$n = \sqrt{1+\chi'}$

at center of line, Er^+ ions do not contribute to n

So, $\boxed{\chi' = n_{glass}^2 - 1 = 1.25}$

$\chi'' = \frac{n\alpha\lambda_0}{2\pi} = \frac{1.5\cdot 10\text{ cm}^{-1}\cdot 980\text{ nm}}{2\pi}$

$\boxed{\chi'' = 2.3\times10^{-4}}$

4. a) Since collisions are slow, $\Delta\nu_{homo} \sim \frac{1}{2\pi} \frac{1}{t_{decay}}$

lower state decays fastest, so

$$\boxed{\Delta\nu_{homo} \sim \frac{1}{2\pi} \frac{1}{10\,ns} \sim 10^7\ Hz}$$

b) Doppler width $\boxed{\Delta\nu_{inhomo} \sim \frac{\Delta v}{\lambda} \sim \frac{1000\frac{m}{s}}{1\,\mu m} \sim 10^9\ Hz}$

5. In steady-state operation, gain = loss.

So, gain per pass = loss per pass = $\boxed{2\%}$

6. Pick reference plane just after mirror 1.

Then round trip matrix is

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{10cm} & 1 \end{bmatrix} \begin{bmatrix} 1 & 10cm \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{10cm} & 1 \end{bmatrix} \begin{bmatrix} 1 & 10cm \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 10 \\ \frac{1}{10} & 2 \end{bmatrix} \begin{bmatrix} 1 & 10 \\ -\frac{1}{10} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 10 \\ -\frac{1}{10} & 1 \end{bmatrix} \qquad AD - BC = 1\ \checkmark$$

6. (cont)

Then at reference plane, $q = \frac{1}{C}\left[\frac{A-D}{2} \pm i\sqrt{1-\left(\frac{A+D}{2}\right)^2}\right]$

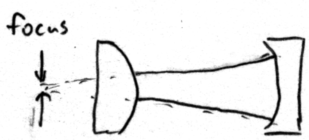
$$= -10\left[\frac{-1}{2} \pm i\sqrt{1-\frac{1}{4}}\right]$$

$$= 5\text{cm} + 10i\sqrt{\frac{3}{4}}\text{ cm}$$

$$\boxed{q = 5\text{cm} + i\,5\sqrt{3}\text{ cm}}$$

$$= z + iz_0$$

a) So, focus lies $\boxed{5\text{cm}}$ behind M_1, and spot size is



$$\boxed{w_0 = \sqrt{\frac{\lambda z_0}{\pi}} = 140\mu\text{m}}$$

b) $\frac{1}{q} = 0.05\text{ cm}^{-1} - i\,0.0866\text{ cm}^{-1}$ at M_1

So, at M_1, $0.0866\text{ cm}^{-1} = \frac{\lambda}{\pi w^2}$

$$\boxed{w = 160\mu\text{m}}$$

at M_2, $q = 15\text{ cm} + i\,5\sqrt{3}\text{ cm}$

$$\frac{1}{q} = 0.05\text{cm}^{-1} - i\,0.0289\text{cm}^{-1}$$

$$\boxed{w = 280\mu\text{m}}$$

7. Expect gain $g_0 = 2\gamma_0 l$

$l = 1\text{ cm}$

$$\gamma_0 = \frac{1}{8\pi} \lambda^2 \Delta N \frac{g(\nu)}{t_s}$$

$$\lambda = \frac{700\text{nm}}{2} = 350\text{nm}$$

$$t_s = 10\text{ns}$$

at peak, $g(\nu) \approx \frac{1}{\Delta\nu} = \frac{1}{10^{13}\text{Hz}}$

Need ΔN: for 4-level scheme,

$$\Delta N = NR(\tau_2 - \tau_1) \approx NR\tau_2$$

$$\tau_2 = 10\text{ns}$$

Also, have $NR_3 = \alpha_3 \frac{I_3}{h\nu_3}$ for 0-3 pumping transition

$I_3 \approx \frac{P_3}{\pi w^2}$ for $w \approx 280\mu\text{m}$ from problem 2.

$P_3 = 8\text{ W}$

$\alpha_3 = 1\text{ cm}^{-1}$

$\nu_3 = \frac{c}{\lambda_3} = \frac{c}{532\text{nm}}$

So, $I_3 = 3.25 \times 10^7 \frac{\text{W}}{\text{m}^2}$ $\left(= 2.83 \times 10^7 \text{ for } w = 300\mu\text{m}\right)$

$$NR_3 = (100\text{ m}^{-1}) \frac{3.25 \times 10^7 \frac{\text{W}}{\text{m}^2} \cdot 532\text{nm}}{6.626 \times 10^{-34}\text{J}\cdot\text{s} \cdot 3 \times 10^8 \frac{\text{m}}{\text{s}}} = 8.7 \times 10^{27} \frac{1}{\text{s m}^3}$$

So, $\Delta N = 8.7 \times 10^{19}\ m^{-3}$

$$\gamma_0 = \frac{1}{8\pi}(350nm)^2(8.7\times10^{19}\,m^{-3})\frac{1}{10ns}\frac{1}{10^{13}Hz}$$

$$= 4.24\ m^{-1} \qquad (= 3.7\ m^{-1} \text{ for } w = 300\mu m)$$

so $\boxed{g_0 = 0.085}$ $\qquad (= 0.074 \text{ for } w = 300\mu m)$

or,

$$g_0 = \frac{l}{4\pi}\lambda^2\frac{1}{\Delta\nu}\frac{\alpha_3\lambda_3}{hc}\frac{P}{\pi w^2} = 0.085$$

8. a) For optimum coupling,

$$P_{out} = \pi w^2 I_s(\sqrt{g_0} - \sqrt{L_i})^2$$

$$L_i = 5\times10^{-3}$$

$$I_s = \frac{4\pi h\nu}{\lambda^2}\frac{t_s}{\tau}\frac{1}{2T_2}$$

$\uparrow$ homogeneous line shape

$$= \frac{2\pi h\nu}{\lambda^2 T_2} = \frac{6.28\times6.626\times10^{-34}J\cdot s\times3\times10^8\,m/s}{(350nm)^2\cdot700nm\cdot10ps}$$

$$= 1.46\times10^6\ W/m^2$$

$$P_{out} = \pi(280\mu m)^2\cdot1.46\times10^6\frac{W}{m^2}\left(\sqrt{0.085}-\sqrt{5\times10^{-3}}\right)^2$$

$$= 0.0174\ W = \boxed{17.4\ mW}$$

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or, if $g_0 = 0.04$, $P_{out} = 6\ mW$

8. (b)

Can have about one mode oscillating per homogeneous line width, $\Delta\nu_{homo} = \frac{1}{\pi T_2}$

$= 32$ GHz

Check that $\Delta\nu_{homo} \gg \Delta\nu_L = \frac{c}{L} = \frac{3\times10^8 \text{ m/s}}{220 \text{ cm}} = 1.5$ GHz ✓

So, expect about $\frac{\Delta\nu_{inhomo}}{\Delta\nu_{homo}} \approx \frac{10^{13}\text{ Hz}}{32\text{ GHz}}$

$\approx \boxed{300 \text{ modes}}$ to oscillate

Then $P_{TOT} \sim 300 \times 17\text{ mW} \sim \boxed{5\text{ W}}$

Note that can never have two modes lasing within frequency range $\Delta\nu_{homo}$, since same molecules will contribute to gain. So, mode with higher gain will saturate, other mode will not oscillate.