Phys 532 Final Exam May 5

9:00 – 12:00 AM

Instructions: This is an in-class exam that must be completed during the allotted 3 hour period. You may use your class notes and the textbook for reference. You may also use a calculator and scratch paper as desired.

The exam consists of five problems, each worth 20 points. (That doesn't necessarily mean that the problems are of equal difficulty.) For all problems, partial credit will be given, so be sure to show your work clearly.

Some constants:

$\hbar = 1.054 \times 10^{-34}$ J s
$m_e = 9.109 \times 10^{-31}$ kg
$e = 1.602 \times 10^{-19}$ C
$\epsilon_0 = 8.859 \times 10^{-12}$ C/Vm
$\mu_0 = 4\pi \times 10^{-7}$ T m/A
$c = 2.998 \times 10^{8}$ m/s

and a useful integral:

$$\int_{-1}^{1} (1-u^2)^{n-\frac{1}{2}} du = \frac{(2n)!}{2^{2n}(n!)^2}\pi \qquad \text{(integer } n \geq 0\text{)}$$

1. Design a transverse electro-optic frequency modulator using KDP. Here "transverse" means that you want the direction of the applied electric field to be perpendicular to the propagation direction of the light. (Since the crystal is usually long and skinny, transverse modulators have a larger electric field for a given applied voltage.) Recall that a frequency modulator is intended to vary the phase of the optical field, but not its polarization. (Phase modulation and frequency modulation are the same thing.)

Given that the index ellipsoid for KDP in an applied field is

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{41}yzE_x + 2r_{52}xzE_y + 2r_{63}xyE_z = 1$$

(a) Determine the optimum orientations of the crystal axes, the light propagation direction, the direction of the applied electric field, and the light polarization. Draw a clear sketch showing your design.
(b) Determine the voltage required to produce a π phase shift in the output field. Use the following data: crystal dimensions are 2 cm $\times$ 2 mm $\times$ 2 mm, light wavelength is 633 nm, indices of refraction are $n_o = 1.51$, $n_e = 1.47$, $r_{41} = r_{52} = 8.6 \times 10^{-12}$ m/V and $r_{63} = 10.6 \times 10^{-12}$ m/V.

2. Suppose a certain laser system has an upper laser level 2 which decays radiatively with lifetime $t_s = 1$ μs and has a coherence time $T_2 = 2t_s$. The transition is inhomogeneously broadened to a linewidth $\Delta\nu = 1$ GHz. The laser cavity itself has a longitudinal mode spacing of $\Delta\nu_L = 100$ MHz and a total loss per round trip of 10%. In steady state CW operation, the laser has a power output of P_0. Estimate the maximum peak power achievable if the laser is run as
(a) a Q-switched pulsed laser, and
(b) a mode-locked pulsed laser.

3. A crystal, "yarivite," is to be used for second harmonic generation. It has symmetry class 6mm, which gives nonlinear elements $d_{15} = d_{24} \approx d_{31} = d_{32}$, along with an independent element d_{33}.

(a) If the indices of refraction for yarivite are

	n_0	n_e
ω	1.7	1.5
2ω	1.8	1.6

then:

(i) sketch an optimal phase matching scheme, showing the orientation of the crystal axes and the light polarizations;

(ii) calculate the phase matching angle;

(iii) find the effective nonlinear coupling d'; include any necessary factors for the symmetry of the incident fields.

(b) Answer the same three questions if the indices of refraction are

	n_0	n_e
ω	1.5	1.7
2ω	1.6	1.8

4. The CW efficiency of second harmonic generation can be improved by placing the nonlinear crystal inside a laser cavity, where light of frequency ω oscillates while light at 2ω is allowed to escape. Suppose this is done using a laser with small signal gain (at ω) of g_0, saturation intensity I_s and cavity loss of L_i per round trip. The crystal used for SHG has a nonlinear conversion efficiency α, such that for incident intensity I^ω the output is $I^{2\omega} = \alpha(I^\omega)^2$. In terms of these parameters, what intensity of light at 2ω will be produced?

5. Suppose you made a "graded index" version of a diode laser using AlGaAs. You cause the percentage of aluminum atoms to vary in a continous way to create a graded double heterostructure as illustrated below, where x is the spatial coordinate across the junction. The conduction and valence band edges vary as

$$E_c(x) = E_c(0) + \alpha x^2$$
$$E_v(x) = E_v(0) - \beta x^2$$

respectively, with $\alpha = 1$ eV/μm^2 and $\beta = 0.5$ eV/μm^2. The aluminum gradient also causes the index of refraction to vary as

$$n(x) = n_0(1 - \frac{1}{2}a^2x^2)$$

with $n_0 = 3.6$ and $a = 0.7\ \mu\text{m}^{-1}$.

(a) Find the beam waist along x of the the Gaussian confined mode for this structure, assuming a (vacuum) laser wavelength of 850 nm.

(b) Suppose there are $\sigma_c = \int N_c(x)dx$ eletrons per unit area in the conduction band, and σ_v holes per area in the valence band. Find the spatial extent (along x) of the regions in which the electrons and holes are respectively confined.

(c) Given that the effective masses of electrons and holes are $m_c = 0.067m_e$ and $m_v = 0.46m_e$, do you expect this laser to work very well? Explain.

Heterojunction structure: