Phys 532 Final Exam Solutions

1.a) To get shift linear in E, need to use E_z term.

Then ellipsoid is

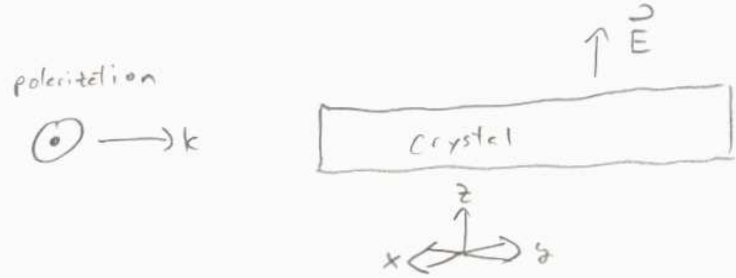
$$\frac{x^2+y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2 r_{63} xy E_z = 1$$

x & y axes mixed: new coefficients are eigenvalues of

$$\begin{bmatrix} \frac{1}{n_o^2} & r_{63} E_z \\ r_{63} E_z & \frac{1}{n_o^2} \end{bmatrix} = \frac{1}{n_o^2} \pm r_{63} E_z$$

and new x' y' axes at 45° to x y

For transverse effect, need $\hat{z} \perp \vec{k}$:



So, want x' or y' also $\perp$ to $\vec{k}$

or, x & y at 45° as shown

Polarize light $\perp$ to z, along x' or y'

1.b) Phase shift $\Delta\phi = kL\Delta n$ $\qquad L = 2\,cm$

$$\Delta n = \frac{1}{2} n_o^3 r_{63} E_z$$

$$= \frac{1}{2} n_o^3 r_{63} \frac{V}{t} \qquad t = 2\,mm$$

$$\Delta\phi = \frac{2\pi}{\lambda} \frac{L}{t} \frac{1}{2} n_o^3 r_{63} V = \pi$$

$$V = \frac{\lambda t}{L} \frac{1}{n_o^3 r_{63}}$$

$$= \frac{633\,nm \cdot 2\,mm}{2\,cm} \cdot \frac{1}{(1.51)^3 (10.6\,\frac{pm}{V})}$$

$$\boxed{V_\pi = 1730\,V}$$

2. For pulsed laser, $P_{pk} \approx P_{cw} \frac{T}{\Delta t}$

T = period
Δt = duration

a) For Q switch, reach equilibrium population after $T \sim t_s$; does no good to wait longer.

and $\Delta t \approx t_c = \frac{1}{\Gamma \Delta\nu_L} = 10^{-7}\,s$

So $P_{pk} = P_o \cdot \frac{t_s}{t_c} = P_o \cdot \frac{10^{-6}\,s}{10^{-7}\,s} = \boxed{10 P_o}$

b) For mode lock, $T = \frac{1}{\Delta\nu_L}$ and $\Delta t = \frac{1}{\Delta\nu}$

$$P_{pk} = P_o \cdot \frac{\Delta\nu}{\Delta\nu_L} = P_o \frac{1\,GHz}{100\,MHz} = \boxed{10 P_o}$$

3. a) ω: 2ω:

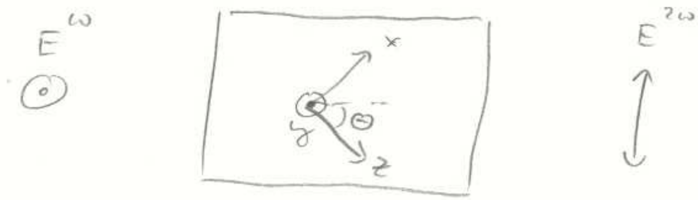
n_o —— —— n_o

n_e —— —— n_e

use $n_o(\omega)$
$n_e(2\omega)$

Have d_{xxz}, d_{yyz}, d_{zxx}, d_{zyy} and d_{zzz}

x and y are the same, so take crystal as



Need 2ω as e, ω as o, as shown

Then using d_{zyy} component, get

$$\boxed{d' = \frac{1}{2} d_{32} \sin\theta}$$

(Get $\frac{1}{2}$ factor from symmetry.)

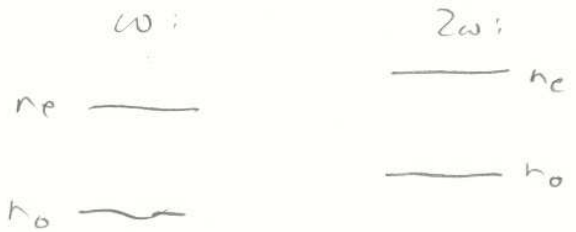
Find θ:
$$\frac{\cos^2\theta}{n_o^{2\omega\,2}} + \frac{\sin^2\theta}{n_e^{2\omega\,2}} = \frac{1}{n_o^{\omega\,2}}$$

$$\cos^2\theta\left(\frac{1}{n_o^{2\omega\,2}} - \frac{1}{n_e^{2\omega\,2}}\right) = \frac{1}{n_o^{\omega\,2}} - \frac{1}{n_e^{2\omega\,2}}$$

$$\cos^2\theta\,(-0.0820) = -0.0446$$

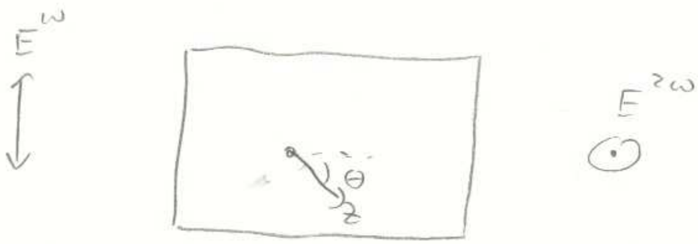
$$\boxed{\theta = 42.5^\circ}$$

3. (b)



Use $n_o(2\omega)$
$n_e(\omega)$

Now have:

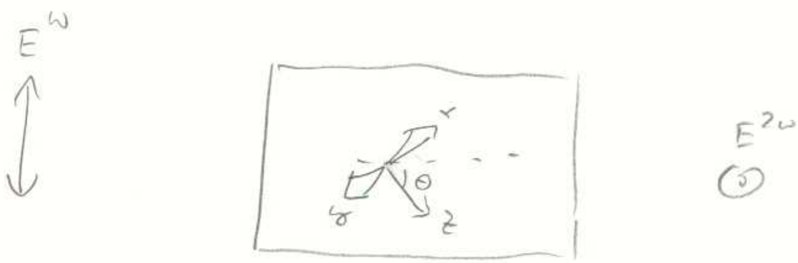


Now have 2ω as o, ω as e

Need to use d_{xxz} or d_{yyz}, since no $E_z^{2\omega}$ component

So, if using $E_x^{2\omega}$, also need E_x^{ω}

Get with 45° rotation:



Then
$$d' = \underset{(E_x^{\omega})}{\frac{1}{\sqrt{2}}} \; \underset{(E_z^{\omega})}{\sin\theta} \; \underset{(E_x^{2\omega})}{\frac{1}{\sqrt{2}}} d_{15} = \boxed{\frac{1}{2} \sin\theta \, d_{15}}$$

[With symmetry factor, same as (a)]

3. (cont)

Find θ:

$$\frac{1}{(n_o^{2\omega})^2} = \frac{\cos^2\theta}{(n_o^{\omega})^2} + \frac{\sin^2\theta}{(n_e^{\omega})^2}$$

$$\cos^2\theta\left[\frac{1}{(n_o^{\omega})^2} - \frac{1}{(n_e^{\omega})^2}\right] = \frac{1}{(n_o^{2\omega})^2} - \frac{1}{(n_e^{\omega})^2}$$

$$\cos^2\theta\left[\,0.0984\,\right] = 0.0446$$

$$\boxed{\theta = 47.7^\circ}$$

4. Know that in operation, gain = loss

$$\text{gain} = \frac{g_0}{1 + I/I_s}$$

here $\text{loss} = L_i + \alpha I$

(not αI^2, since loss = fraction lost per round trip)

So

$$\frac{g_0}{1 + \frac{I}{I_s}} = L_i + \alpha I$$

$$g_0 = L + \alpha I + \frac{L}{I_s} I + \frac{\alpha}{I_s} I^2$$

$$I^2 + \left(I_s + \frac{L}{\alpha}\right) I + \frac{I_s}{\alpha}(L - g_0) = 0$$

$$I = \frac{1}{2}\left(-I_s - \frac{L}{\alpha} \pm \sqrt{\left(I_s + \frac{L}{\alpha}\right)^2 - 4\frac{I_s}{\alpha}(L - g_0)}\right)$$

To get $I > 0$, need $+$ sign,
and need $g_0 > L$

$$I_{cavity} = \frac{1}{2}\left[\sqrt{\left(I_s + \frac{L}{\alpha}\right)^2 + 4\frac{I_s}{\alpha}(g_0 - L)} - \left(I_s + \frac{L}{\alpha}\right)\right]$$

$$= \frac{1}{2}\left[\sqrt{\left(I_s - \frac{L}{\alpha}\right)^2 + 4I_s\frac{g_0}{\alpha}} - \left(I_s + \frac{L}{\alpha}\right)\right]$$

This is light at ω oscillating in cavity,

so $I^{2\omega} = \alpha I^2$

$$= \frac{\alpha}{4}\left[\left(I_s - \frac{L}{\alpha}\right)^2 + 4I_s\frac{g_0}{\alpha} + \left(I_s + \frac{L}{\alpha}\right)^2 - 2\left(I_s + \frac{L}{\alpha}\right)\sqrt{\left(I_s - \frac{L}{\alpha}\right)^2 + 4I_s\frac{g_0}{\alpha}}\right]$$

$$= \frac{\alpha}{4}\left[2\left(I_s^2 + \frac{L^2}{\alpha^2}\right) + 4I_s\frac{g_0}{\alpha} - 2\sqrt{\left(I_s^2 + \frac{L^2}{\alpha^2}\right)^2 + 4I_s\frac{g_0}{\alpha}\left(I_s + \frac{L}{\alpha}\right)^2}\right]$$

$$\boxed{I^{2\omega} = \frac{1}{2}\left[\alpha I_s^2 + \frac{L^2}{\alpha} + 2g_0 I_s - \sqrt{\left(\alpha I_s^2 + \frac{L^2}{\alpha}\right)^2 + 4\alpha I_s g_0\left(I_s + \frac{L}{\alpha}\right)^2}\right]}$$

5. a) From notes, $w_0 = \sqrt{\frac{\lambda_0}{n_0 \pi a}}$

$$= \sqrt{\frac{850\,nm}{3.6 \cdot \pi \cdot 0.7\,\mu m^{-1}}} = 0.330\,\mu m$$

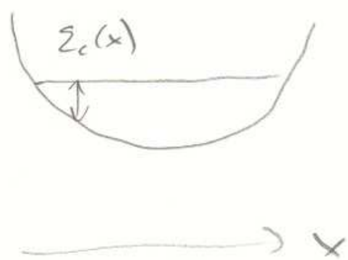
$$\simeq 330\,nm$$

b) Have $\sigma_c = \int N_c(x)\,dx$

At position x, have $\mathcal{E}_{Fc}(x) = \frac{\hbar^2}{2m_c}\left[3\pi^2 N_c(x)\right]^{2/3}$

So $N_c(x) = \frac{1}{3\pi^2}\left[\frac{2m_c}{\hbar^2}\mathcal{E}_c(x)\right]^{3/2}$

where $\mathcal{E}_c(x)$ is energy of Fermi level above band edge:



So, $\mathcal{E}_c(x) = \mathcal{E}_c(0) - \alpha x^2$

If we determine $\mathcal{E}_c(0)$, then find size of region by finding where $\mathcal{E}_c(x) \to 0$

Say $\varepsilon_c(x) = 0$ at $x = x_c$

Then $\varepsilon_c(0) = \alpha x_c^2$

So, have

$$\sigma_c = \int_{-x_c}^{x_c} \frac{1}{3\pi^2}\left[\frac{2m_c}{\hbar^2}\left(\alpha x_c^2 - \alpha x^2\right)\right]^{3/2} dx$$

Solve for x_c

$$\sigma_c = \frac{1}{3\pi^2}\left(\frac{2m_c\alpha}{\hbar^2}\right)^{3/2} \int_{-x_c}^{x_c} \left(x_c^2 - x^2\right)^{3/2} dx$$

$$= \frac{1}{3\pi^2}\left(\frac{2m_c\alpha}{\hbar^2}\right)^{3/2} x_c^4 \underbrace{\int_{-1}^{1} (1-u^2)^{3/2} du}$$

$n = 2$, so

$$= \frac{4!}{2^4(2!)^2}\pi$$

$$= \frac{24}{64}\pi = \frac{3}{8}\pi$$

$$\sigma_c = \frac{1}{8\pi}\left(\frac{2m_c\alpha}{\hbar^2}\right)^{3/2} x_c^4$$

$$\boxed{x_c = (8\pi\sigma_c)^{1/4}\left(\frac{\hbar^2}{2m_c\alpha}\right)^{3/8}}$$

Expect same calculation for v band,

$$\text{So} \quad x_v = (8\pi\sigma_v)^{1/4}\left(\frac{\hbar^2}{2m_v\beta}\right)^{3/8}$$

c) Problem because charge neutrality requires

$$\sigma_c = \sigma_v$$

But $\quad m_v\beta = 0.46 \cdot 0.6 \ \frac{eV\, m_e}{\mu m^2}$

$\qquad = 0.28 \ \frac{eV\, m_e}{\mu m^2}$

while $\quad m_c\alpha = 0.067 \cdot 1 \ \frac{eV\, m_e}{\mu m^2}$

So, $\quad \frac{x_c}{x_v} = \left(\frac{0.28}{0.067}\right)^{3/8} = 1.7$

$\boxed{\text{holes and electrons don't overlap very well, can't match both to mode.}}$

If $x_c = w_0$, need

$$\sigma_c = \frac{1}{8\pi}\left(\frac{2m_e\alpha}{\hbar^2}\right)^{3/2} w_0^4$$

$$= \frac{1}{8\pi}\left(\frac{2\cdot 0.067\cdot 9.1\times10^{-31}\,\text{kg}\cdot \frac{1.6\times10^{-19}\,\text{J}}{\mu m^2}}{(1.054\times10^{-34}\,\text{J}\cdot\text{s})^2}\right)^{3/2}\cdot(250\,\text{nm})^4$$

$= 3.6\times10^{17}\ \text{m}^{-2}$ so $J = 1.4\times10^{7}\ \frac{A}{m^2}$

$I \sim 35\,\text{mA}$ at $500\,\mu m \times 50\,\mu m$