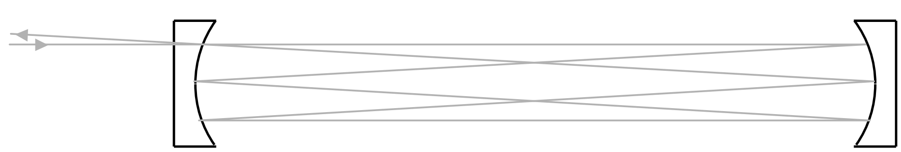
Phys 532 Assignment 1 Due January 24

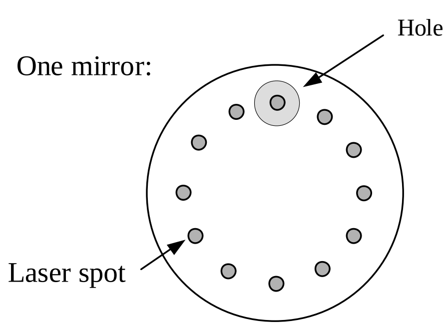
1. (a) Use Snell's Law and the law of reflection to derive the ray transfer matrices of Yariv Table 6.1, items (3), (4), and (5).
(b) Using these matrices, calculate the focal length of a plano-convex lens constructed of a material having index of refraction n and radius of curvature R. (A plano-convex lens has one surface flat and on surface curved inward.)

2. Show that for an arbitrary optical system of lenses and mirrors, the system transfer matrix satisfies $AD - BC = n_1/n_2$, where n_1 and n_2 are the indices of refraction at the input and output planes, respectively.

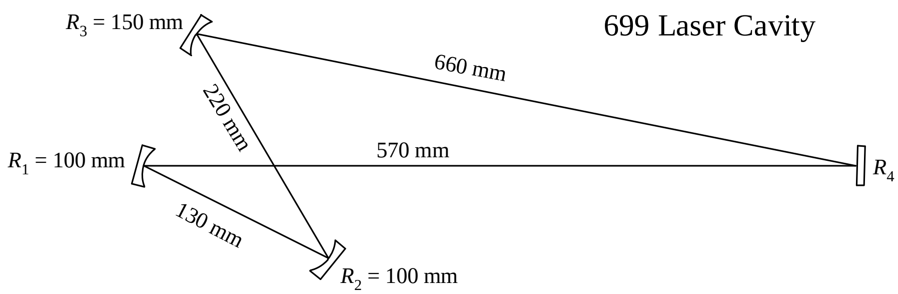
3. A White cell is an optical cavity consisting of two mirrors, one of which has a small hole through which light can pass unimpeded. It is possible to introduce a laser beam through the hole such that it bounces back and forth between the two mirrors many times before exiting. This is often used for spectroscopic applications to give an increased interaction length.

Suppose a White cell is to be constructed with identical mirrors separated by 30 cm. The beam should hit each mirror twelve times before exiting, as illustrated. Find the minimum allowed value for the radius of curvature of the mirrors.





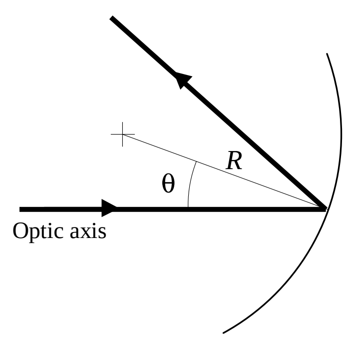
4. The Model 699 laser from the Coherent Laser Corporation is a typical commercial ring laser. The cavity in this laser consists of four mirrors in a "bowtie" configuration, as sketched below. Determine the range of values for R_4 such that the cavity is optically stable. You may assume that the angles of incidence on the mirrors are small enough to be negligible. (Suggestion: calcuate the required matrices using a computer or scientific calculator.)



5. Suppose a Gaussian laser beam with $\lambda = 532$ nm is collimated with a spot size of 50 μm at the point $z = 0$. If a a thin lens with focal length 25 mm is placed at $z = 35$ mm, find the position and spot size for the resulting focus. Compare the focal postition with that predicted by geometrical optics.

822 students only:

6. Show that for rays striking a mirror at an oblique angle θ as shown, the ray transfer matrices are



$$M_{\parallel} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{R\cos\theta} & 1 \end{bmatrix}$$

for the direction in the plane of the picture, and

$$M_{\perp} = \begin{bmatrix} 1 & 0 \\ -\frac{2\cos\theta}{R} & 1 \end{bmatrix}$$

for the direction normal to the picture. Here R is the radius of curvature of the spherical mirror. You need only calulate the nontrivial C element in the lower left corner.

One approach to this problem is to use the vector form of the law of reflection,

$$\hat{\mathbf{v}}_{\text{out}} = \hat{\mathbf{v}}_{\text{in}} - 2(\hat{\mathbf{n}} \cdot \hat{\mathbf{v}}_{\text{in}})\hat{\mathbf{n}}$$

where $\hat{\mathbf{v}}_{\text{in}}$ is the direction vector of the incident ray, $\hat{\mathbf{v}}_{\text{out}}$ is the direction vector of the reflected ray, and $\hat{\mathbf{n}}$ is a unit vector normal to the reflecting surface. Use this law to determine how an input ray parallel to the optic axis will be reflected. The angle between the resulting output ray and the (reflected) optic axis gives you the desired matrix element.