Phys 532 Assignment 10 Due April 11

1. In Section 16.3, Yariv presents a simple classical model to explain the origin of nonlinear optical effects. Work through a similar model as follows:

(a) Suppose a "classical" electron moves in a 1-dimensional potential

$$V(x) = V_0 \left(\frac{1}{2}\frac{x^2}{a^2} + \frac{1}{3}\frac{x^3}{a^3} \right)$$

where V_0 is a characteristic atomic energy scale and a is a characteristic atomic length scale. Write out the classical equation of motion for an electron of mass m in this potential which is also driven by an electric field $E(t) = E_0 \exp(i\omega t)$. What is the resonant frequency ω_0 in the limit of small excursion x?

(b) Assuming a steady-state solution of the form

$$x(t) = x_1 e^{i\omega t} + x_2 e^{2i\omega t} + x_3 e^{3i\omega t} + \ldots$$

solve for the amplitudes of the first three terms x_1, x_2, and x_3. These terms will contribute to the polarization, and thus give the first, second, and third order susceptibilities.

(c) For most optical materials, the lowest resonant frequencies are in the ultraviolet, so we can take $\omega \ll \omega_0$. Evaluate the amplitudes in this limit, and show that

$$\frac{x_1}{a} \approx \frac{eaE_0}{V_0}$$

and $x_2/x_1 \approx x_3/x_2 \approx x_1/a$. So, as long as $eE_0 \ll V_0/a$, the terms in the expansion of x will be progressively decreasing.

2. The fundamental equation for second-order nonlinear response is

$$P(t) = 2dE(t)^2$$

for real electric field E and polarization P. (Assume here that the fields can be treated as scalars.) In class, we showed that for a single applied field oscillating at frequency ω, the complex amplitude of the polarization component at 2ω is

$$P^{2\omega} = d(E^{\omega})^2$$

for complex field amplitude E^{ω}.

(a) Show that if $E(t) = E_1(t) + E_2(t)$ with

$$E_i(t) = \frac{1}{2}\left(E^{\omega_i} e^{i\omega_i t} + E^{\omega_i *} e^{-i\omega_i t} \right)$$

and we are interested in the component of P oscillating at $\omega_3 = \omega_1 + \omega_2$, then the complex amplitudes E_1, E_2 and P are related as

$$P^{\omega_3} = 2dE^{\omega_1}E^{\omega_2}.$$

(b) Suppose now that $\omega_1 = \omega_2 + \epsilon$. Show that on time scales $t \ll 1/\epsilon$, the polarization becomes $P^{\omega_3} = dE^2$ for $E = E^{\omega_1} + E^{\omega_2}$, just as would be obtained if $\omega_1 = \omega_2$. This should illustrate the continuity between the degenerate and non-degenerate cases.

3. Design a second-harmonic generation experiment in Te using an input at $\lambda = 10.6\ \mu$m. Te has point-group symmetry 32. Find the index-matching angle and decide on the proper beam polarization and crystal orientation for maximum power output at 5.3 μm. Find the effective value of the nonlinear coupling parameter, d'. You can obtain the necessary indices of refraction by interpolating from the table on Yariv pg 405.

4. Look up the properties of the nonlinear crystal β-BaB_2O_4, commonly called BBO. Find its transmission range, symmetry class, and values for its nonzero nonlinear coefficients. Also find its index of refraction at 1064 nm and 532 nm. Cite the source you use.