Phys 532 Assignment 11 Due April 18

1. Suppose a particular second-harmonic generation arrangment is run with CW input power P^ω and output power $P^{2\omega}$ such that the conversion efficiency is $P^{2\omega}/P^\omega = \epsilon \ll 1$. Now drive it instead with pulsed laser light having a pulse duration of Δt and repetition rate T. If the *average* input power is still the same P^ω, find the new value of the conversion efficiency. (Note that once $\epsilon \sim 1$, the approximation that the field at ω is undepleted breaks down. Assume ϵ small enough that you don't have to account for this.)

2. As mentioned in class, $LiNbO_3$ can be used with noncritical phase matching to frequency-double light with a wavelength of 1064 nm. Suppose instead you wish to double 1100 nm light to a wavelength of 550 nm. Dmitriev, Gurzadyan and Nikogosyan list the indices of refraction for $LiNbO_3$ as

$$n_o^2 = 4.9130 + \frac{0.1173 + 1.65 \times 10^{-8}T^2}{\lambda^2 - (0.212 + 2.7 \times 10^{-8}T^2)^2} - 0.0278\lambda^2$$

and

$$n_e^2 = 4.5567 + 2.605 \times 10^{-7}T^2 + \frac{0.0970 + 2.70 \times 10^{-8}T^2}{\lambda^2 - (0.201 + 5.4 \times 10^{-8}T^2)^2} - 0.0224\lambda^2$$

at temperature T in K and wavelength λ in μm. Given that $LiNbO_3$ has a nonlinear coefficient $d_{31} = 4 \times 10^{-23}$ C/V^2, find:
(a) The temperature required for noncritical phase matching
(b) The optimum output power obtained for input power of 100 mW and a 3-cm long crystal.

3. A tunable laser source in the infrared can be obtained by difference-frequency generation between two diode lasers. Suppose diodes at $\lambda_1 = 680$ nm and $\lambda_2 = 808$ nm are used to generate light at $\lambda_3 = 4.3$ μm, using the crystal proustite (Ag_3AsS_3). Its symmetry group is 3m, and the relevant indicies of refraction are

$\lambda(\mu\text{m})$	n_o	n_e
0.680	2.9685	2.6996
0.808	2.8865	2.6366
4.3	2.7347	2.5203

(a) If each diode can be tuned by ± 3 nm, what range of wavelengths in the infrared can be obtained?
(b) Suppose a phase matching scheme uses the $d_{16} = -18$ pm/V nonlinear coefficient, along with an extraordinary ray at λ_1, an ordinary ray at λ_2, and an extraordinary ray at λ_3. Draw a sketch showing the light polarizations and crystal axes for this case.
(c) Determine the phase matching angle θ_m between the crystal z axis and the light propagation direction. What is the effective nonlinear coupling d'?

Phys 822 students only:

4. An important limitation of second-harmonic generation is beam walk-off. This occurs because when a wave propagates through an isotropic crystal in a direction which is not a symmetry axis, the direction of the phase velocity $\mathbf{k}$ is not the same as the direction of energy flow $\hat{\mathbf{s}} = \mathbf{E} \times \mathbf{H}/(|\mathbf{E}||\mathbf{H}|)$. Phase matching occurs when $\mathbf{k}^{2\omega} = 2\mathbf{k}^{\omega}$, which means that $\hat{\mathbf{s}}^{2\omega} \neq \hat{\mathbf{s}}^{\omega}$. The fundamental and harmonic beams will therefore diverge as they propagate, which limits the length of crystal that can be effectively used.

Suppose a beam is propagating through a uniaxial crystal in the direction

$$\hat{\mathbf{k}} = \sin\theta\hat{\mathbf{x}} + \cos\theta\hat{\mathbf{z}}$$

where the index of refraction along x is n_o and along z is n_e. Then if the beam is polarized in the $x - z$ plane, it will have an effective index of refraction

$$\frac{1}{n^2} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}.$$

Also, it can be shown that the ratio of the x and z components of the electric field will be

$$\frac{E_x}{E_z} = \frac{\sin\theta(n^2 - n_e^2)}{\cos\theta(n^2 - n_o^2)}.$$

(This is essentially the uppermost equation on Yariv pg. 90.)

(a) Calculate the "walk-off" angle β, using $\cos\beta = \hat{\mathbf{s}} \cdot \hat{\mathbf{k}}$. To evaluate $\hat{\mathbf{s}}$, you can take $\mathbf{H} \propto \hat{\mathbf{y}}$.

(b) To obtain a more useful expression for small β, evaluate $\sin\beta = (1 - \cos^2\beta)^{1/2}$. I get

$$\sin\beta = \frac{\sin\theta\cos\theta(n_e^2 - n_o^2)}{\sqrt{n_e^4\cos^2\theta + n_o^4\sin^2\theta}}$$

(c) Calculate β for the KDP example we treated in class and that Yariv treats on pages 395–396.