1. Suppose an optical parametric oscillator is constructed using a $1-\mathrm{cm}$ long BBO crystal that is pumped by the third harmonic of a Nd:YAG laser at 355 nm . The crystal angle is set to $\theta=37^{\circ}$, which is the phase matching angle for outputs at $\lambda_{1}=590 \mathrm{~nm}$ and $\lambda_{2}=890 \mathrm{~nm}$. This is a type I phase matching scheme using ordinary output beams and the $d_{21}=2 \times 10^{-23} \mathrm{C} / \mathrm{V}^{2}$ nonlinear element. The $\lambda_{1}$ light oscillates in a cavity with an internal loss of $L_{i}=5 \%$ per round trip and an output coupling transmission $T=5 \%$. Assuming the cavity mode has an optimum focal spot size, find the threshold pump power. If a pulsed pump laser with peak power of 1 kW is used, estimate the power at $\lambda_{1}$ produced. The data you need for this and problem 2 are: $n_{1 o}=1.671, n_{2 o}=1.660, n_{3 o}=1.720, n_{3 e}=1.586$, $d n_{1} / d \lambda \approx d n_{2} / d \lambda=3.3 \times 10^{-5} \mathrm{~nm}^{-1}$.
2. The output frequency of an OPO can be tuned by adjusting the crystal angle $\theta$. An expression for the tuning sensitivity $d \lambda_{1} / d \theta$ can be found through the following argument:
(a) Phase matching requires that

$$
\begin{equation*}
n_{3} \omega_{3}=n_{1} \omega_{1}+n_{2} \omega_{2} \tag{1}
\end{equation*}
$$

where the pump frequency $\omega_{3}$ is fixed and the output frequencies $\omega_{1}$ and $\omega_{2}$ can vary. Assume that the output beams are both ordinary, so that only $n_{3}$ depends explictly on $\theta$. However, $n_{1}$ and $n_{2}$ do depend on $\omega_{1}$ and $\omega_{2}$, which themselves change with $\theta$. Show then that taking the $\theta$ derivative of (1) yeilds

$$
\omega_{3} \frac{d n_{3}}{d \theta}=\left[n_{1}-n_{2}+\omega_{1} \frac{\partial n_{1}}{\partial \omega}-\omega_{2} \frac{\partial n_{2}}{\partial \omega}\right] \frac{d \omega_{1}}{d \theta}
$$

(b) Show now that

$$
\frac{d n_{3}}{d \theta}=\frac{n_{3}^{3}}{2}\left(n_{3 o}^{-2}-n_{3 e}^{-2}\right) \sin 2 \theta
$$

(c) Combine these results to get an expression for $d \omega_{1} / d \theta$.
(d) For the BBO setup of problem 1, what change in $\theta$ is required to change $\lambda_{1}$ by 10 nm?
3. Consider spontaneous parametric fluorescence in a crystal of $\mathrm{LiNbO}_{3}$, as sketched below. Here many different frequencies are produced, with light of wavelength $\lambda$ propagating at an angle $\theta$ determined by the phase matching conditions. Suppose the crystal is oriented as shown, with a pump at $\lambda_{3}=532 \mathrm{~nm}$ polarized along the crystal $z$ axis. Determine the output angle and polarization for light of wavelength $2 \mu \mathrm{~m}$. Dmitriev, Gurzadyan and Nikogosyan list the indices of refraction for $\mathrm{LiNbO}_{3}$ as

$$
n_{o}^{2}=4.9130+\frac{0.1173+1.65 \times 10^{-8} T^{2}}{\lambda^{2}-\left(0.212+2.7 \times 10^{-8} T^{2}\right)^{2}}-0.0278 \lambda^{2}
$$

and

$$
n_{e}^{2}=4.5567+2.605 \times 10^{-7} T^{2}+\frac{0.0970+2.70 \times 10^{-8} T^{2}}{\lambda^{2}-\left(0.201+5.4 \times 10^{-8} T^{2}\right)^{2}}-0.0224 \lambda^{2}
$$

at temperature $T$ in K and wavelength $\lambda$ in $\mu \mathrm{m}$.


