1. Suppose an optical parametric oscillator is constructed using a 1-cm long BBO crystal that is pumped by the third harmonic of a Nd:YAG laser at 355 nm. The crystal angle is set to $\theta = 37^{\circ}$, which is the phase matching angle for outputs at $\lambda_1 = 590$ nm and $\lambda_2 = 890$ nm. This is a type I phase matching scheme using ordinary output beams and the $d_{21} = 2 \times 10^{-23} \text{ C/V}^2$ nonlinear element. The λ_1 light oscillates in a cavity with an internal loss of $L_i = 5\%$ per round trip and an output coupling transmission T = 5%. Assuming the cavity mode has an optimum focal spot size, find the threshold pump power. If a pulsed pump laser with peak power of 1 kW is used, estimate the power at λ_1 produced. The data you need for this and problem 2 are: $n_{1o} = 1.671$, $n_{2o} = 1.660$, $n_{3o} = 1.720$, $n_{3e} = 1.586$, $dn_1/d\lambda \approx dn_2/d\lambda = 3.3 \times 10^{-5} \text{ nm}^{-1}$.

2. The output frequency of an OPO can be tuned by adjusting the crystal angle θ . An expression for the tuning sensitivity $d\lambda_1/d\theta$ can be found through the following argument:

(a) Phase matching requires that

$$n_3\omega_3 = n_1\omega_1 + n_2\omega_2 \tag{1}$$

where the pump frequency ω_3 is fixed and the output frequencies ω_1 and ω_2 can vary. Assume that the output beams are both ordinary, so that only n_3 depends explicitly on θ . However, n_1 and n_2 do depend on ω_1 and ω_2 , which themselves change with θ . Show then that taking the θ derivative of (1) yields

$$\omega_3 \frac{dn_3}{d\theta} = \left[n_1 - n_2 + \omega_1 \frac{\partial n_1}{\partial \omega} - \omega_2 \frac{\partial n_2}{\partial \omega} \right] \frac{d\omega_1}{d\theta}$$

(b) Show now that

$$\frac{dn_3}{d\theta} = \frac{n_3^3}{2} \left(n_{3o}^{-2} - n_{3e}^{-2} \right) \sin 2\theta$$

(c) Combine these results to get an expression for $d\omega_1/d\theta$.

(d) For the BBO setup of problem 1, what change in θ is required to change λ_1 by 10 nm?

3. Consider spontaneous parametric fluorescence in a crystal of LiNbO₃, as sketched below. Here many different frequencies are produced, with light of wavelength λ propagating at an angle θ determined by the phase matching conditions. Suppose the crystal is oriented as shown, with a pump at $\lambda_3 = 532$ nm polarized along the crystal z axis. Determine the output angle and polarization for light of wavelength 2 μ m. Dmitriev, Gurzadyan and Nikogosyan list the indices of refraction for LiNbO₃ as

$$n_o^2 = 4.9130 + \frac{0.1173 + 1.65 \times 10^{-8} T^2}{\lambda^2 - (0.212 + 2.7 \times 10^{-8} T^2)^2} - 0.0278\lambda^2$$

and

$$n_e^2 = 4.5567 + 2.605 \times 10^{-7} T^2 + \frac{0.0970 + 2.70 \times 10^{-8} T^2}{\lambda^2 - (0.201 + 5.4 \times 10^{-8} T^2)^2} - 0.0224\lambda^2$$

at temperature T in K and wavelength λ in μ m.

