Phys 532 Assignment 2 Due January 31

1. Suppose that when a Gaussian laser beam passes through an optical system, its complex beam radius is modified according to

$$q_{\rm out} = \frac{A_1 q_{\rm in} + B_1}{C_1 q_{\rm in} + D_1},$$

while a second system modifies $q$ as

$$q_{\rm out} = \frac{A_2 q_{\rm in} + B_2}{C_2 q_{\rm in} + D_2}.$$

Show that a beam passing consecutively through the two systems will have

$$q_{\rm out} = \frac{A_T q_{\rm in} + B_T}{C_T q_{\rm in} + D_T}$$

with

$$\begin{bmatrix} A_T & B_T \\ C_T & D_T \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}$$

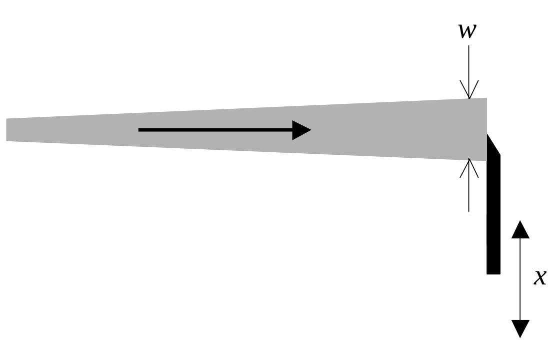
This establishes the correspondence between ray matrices and Gaussian beam propagation.

2. Suppose a Gaussian laser beam with $\lambda = 532$ nm is collimated with a spot size of 50 $\mu$m at the point $z = 0$. If a a thin lens with focal length 25 mm is placed at $z = 35$ mm, find the position and spot size for the resulting focus. Compare the focal position with that predicted by geometrical optics.

3. One way to determine the beam waist $w$ of a Gaussian beam is to intercept the beam with a razor blade (or other straight edge) which is mounted on a mechanical translation stage. Suppose the blade is found to block 30% of the incident power at position $x_1$, and 70% of the power at position $x_2$. Calculate $w$ in terms of the difference $\Delta = x_2 - x_1$. Hint: the error function erf($x$) defined by

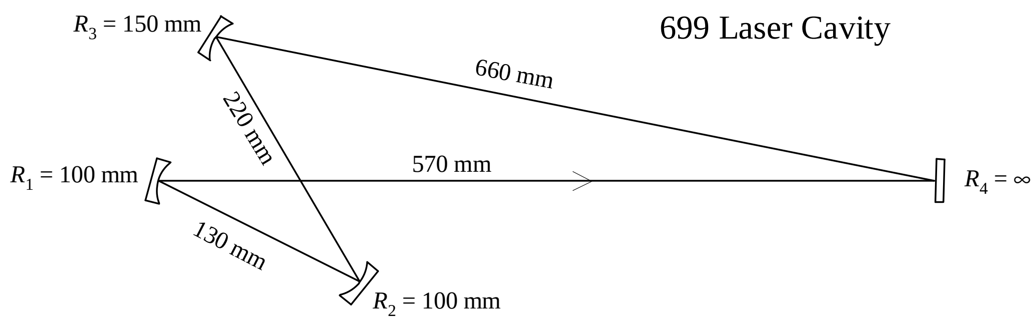
$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

satisfies erf(0.37) = 0.4.



4. What focal length lens should be used to focus a collimated Gaussian beam to a spot a distance $d$ away? Express the answer in terms of the the incident beam waist $w$. What is the largest possible value of $d$ for a given $w$?

5. (a) Find the focus positions and spot sizes of the Gaussian mode supported by the Coherent 699 laser cavity.
(b) In this laser, mirror 4 is only partially reflecting, to allow a fraction of the circulating light to escape. If the beam circulates around the cavity in the sense shown, find the far-field divergence angle of the output beam.




6. What is the longitudinal mode spacing of the 699 cavity, in MHz?

*822 students only:*

7. As Yariv discusses in Section 6.9, higher-order Gaussian beam solutions to the wave equation can also be found. Observation of Eq. (6.9-1) shows that the field distributions for these modes seem to be related to the harmonic oscillator wave functions encountered in quantum mechanics. This relationship can be motivated by looking for solutions to the paraxial wave equation

$$\nabla_t^2\psi - 2ik\frac{\partial\psi}{\partial z} = 0$$

that have the form

$$\psi(x, y, z) = f(x, y)N(z)e^{-\alpha(z)r^2}.$$

Show that at any fixed $z$, the differential equation for $\psi(x, y)$ at that $z$ is formally equivalent to that of the 2D quantum harmonic oscillator.