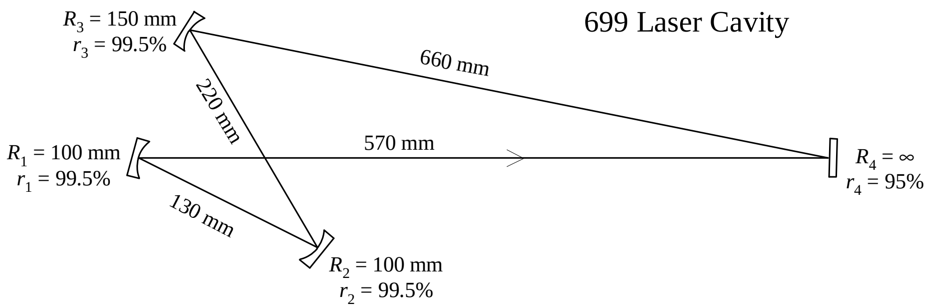
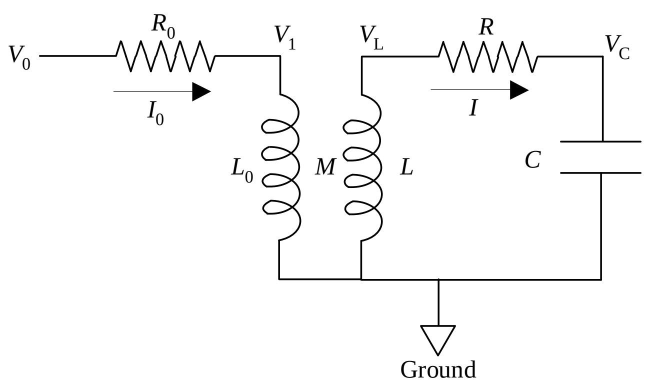
Phys 532 Assignment 3 Due February 7

1. If the mirrors of a Coherent 699 laser cavity have the reflectances r_i shown, and there is an additional 2% loss per pass due to intracavity elements, what is the full-width at half max of the cavity modes?



2. (4 points) An analogy can be drawn between the resonances of an optical cavity and the resonance of the RLC circuit shown. The inductor L and capacitor C form an oscillator, while the resistor R induces losses. Energy is coupled in and out through a transformer as shown, which represents a partially transmitting mirror of the optical system. The transformer has mutual inductance M, so that $V_1 = L_0\dot{I}_0 + M\dot{I}$ and $V_L = -L\dot{I} - M\dot{I}_0$. You may also recall that $\dot{V}_C = I/C$, and $V_L - V_C = IR$. Define $\omega_0^2 = 1/LC$ and $\gamma = R/L$, and take $\gamma \ll \omega_0$.



(a) Assume first that $M = 0$, and calculate the current I in the oscillator circuit for initial conditions $I = I_1$ and $dI/dt = 0$.

(b) From your solution for (a), calculate the quality factor of the oscillator, using

$$Q = \frac{\omega_0 \mathcal{E}}{|d\mathcal{E}/dt|}$$

where

$$\mathcal{E} = \frac{1}{2}CV_C^2 + \frac{1}{2}LI^2$$

is the energy stored in the circuit.

(c) Now take $M \neq 0$ and calculate the steady state current I in the oscillator for an applied voltage $V_0 = A\exp(i\omega t)$, assuming $|\omega - \omega_0| \ll \omega_0$ and $R_0 \gg \omega_0 L_0$. Show that if $\omega_0^2 M^2 \ll R R_0$, the energy transferred to the oscillator has a resonance with full-width at half max given by

$$\Delta\omega_{1/2} = \frac{\omega_0}{Q}$$

3. Suppose a medium has an electric susceptibility $\chi = \chi' - i\chi''$ with $\chi'' \ll 1$. Calculate the real index of refraction n and the absorption coefficient α, where α is defined such that $I(z) = I_0 \exp(-\alpha z)$ for a beam of intensity I propagating through the medium along z.

4. (4 points) Consider a "classical" model of an atom consisting of a nucleus and a single valence electron, both taken to be classical point particles. Assume that the nucleus is fixed in space and that the electron is attached to it by a spring-like force such that its natural oscillation frequency is ω_0. (This force acts in place of the usual Coulomb interaction.)

(a) Calculate the rate γ_R at which the energy of the electron will decay due to radiative damping. Use the Larmor formula for the total power radiated by an accelerating particle,

$$P_{\text{rad}} = \frac{e^2}{6\pi\epsilon_0 c^3}|\dot{v}|^2,$$

where e is the charge and v the velocity, assumed to be non-relativistic.

(b) Write down the differential equation obeyed by the microscopic dipole moment p in the presence of an applied field $E(t)$.

(c) Suppose N atoms are scattered randomly through a volume V. Given a macroscopic polarization $P(t)$, what is the rate at which power is radiated by the sample? If all the atoms are initially excited with the same moment $p(0)$ and they continue to oscillate in phase, calculate $P(t)$ in the absence of a driving field.

(d) Suppose that, in addition to radiative damping, each atom loses energy at a rate γ_1 through some additional form of friction. Suppose also that an elastic collision process occurs at a rate γ_2 per atom, such that when an atom collides, that phase of its oscillation is randomized. Calculate $P(t)$ for a uniform excitation, as in (c).

(e) Write down the differential equation obeyed by P, including the driving field and the losses of (d). Find the steady-state solution P_0 in the presence of a field $E(t) = E_0 \exp(i\omega t)$. What are the electric susceptibility components χ' and χ''?

You may assume that γ_R, γ_1, γ_2 and $|\omega - \omega_0|$ are all small compared to ω_0 throughout the problem.

822 students only:

5. Prove the following properties for a general density matrix ρ:

(a) The eigenvalues of ρ are all between 0 and 1.

(b) $\rho^2 = \rho$ if and only if ρ represents a pure state.

Also, what is the density matrix for an ensemble of atoms, half of which are in state 1 and half in a superposition of states 1 and 2? Verify conditions (a) and (b) for this case.