Phys 532 Assignment 4 Due February 14

1. (4 pts) In class, we gave general expressions for the real and imaginary part of the electrical susceptibility (also Yariv 8.1-19)

$$\chi' = \frac{\mu^2}{\epsilon_0 \hbar} \Delta N_0 \frac{-\Delta T_2^2}{1 + \Delta^2 T_2^2 + 4\Omega^2 T_2 \tau}$$

and

$$\chi'' = \frac{\mu^2}{\epsilon_0 \hbar} \Delta N_0 \frac{T_2}{1 + \Delta^2 T_2^2 + 4\Omega^2 T_2 \tau}$$

Fill in the details of this derivation as follows:
(a) If the Hamiltonian for the two-level system is

$$H = \hbar \begin{bmatrix} 0 & -2\Omega \cos \omega t \\ -2\Omega \cos \omega t & \omega_0 \end{bmatrix}$$

evaluate the coherent evolution equation

$$\left.\frac{d\rho}{dt}\right|_{\text{coh}} = \frac{i}{\hbar}[\rho, H].$$

Here ω_0 is the transition frequency and $\Omega \equiv \mu E_0 / 2\hbar$ for an incident field amplitude E_0 and transition dipole matrix element μ. (Don't try to solve the differential equations at this point, just write them out.)
(b) For the incoherent evolution, take

$$\left.\frac{d\rho}{dt}\right|_{\text{inc}} = \begin{bmatrix} -\Gamma_1 \rho_{11} + \Gamma_2 \rho_{22} & -\gamma \rho_{12} \\ -\gamma \rho_{21} & -\Gamma_2 \rho_{22} + R \end{bmatrix}$$

where Γ_1 is the decay rate out of state 1, Γ_2 is the decay rate $2 \rightarrow 1$, γ is the dephasing rate, and R represents the rate at which molecules are pumped into state 2. Using this matrix, solve for the population difference ΔN_0, defined to be the steady state value of $N(\rho_{11} - \rho_{22})$ in the absence of a driving field (ie, when $\Omega = 0$).
(c) Using the rotating wave approximation, find the steady state solution for the density matrix including both coherent and incoherent evolution. (Note that $\rho_{11} + \rho_{22} \neq 1$ here.) Express the result for σ_{12} in terms of ΔN_0 rather than R.
(d) Using the microscopic dipole moment $\langle p \rangle = \mu(\rho_{12} + \rho_{21})$, find the macroscopic polarization, P, and from it the electric susceptibility components, defined by

$$P = \text{Re}\,\{\epsilon_0 (\chi' - i\chi'') E_0 \exp(i\omega t)\}$$

By setting $\gamma = 1/T_2$ and $\Gamma_2 = 1/2\tau$, you should obtain the results cited above. (In general, the precise definition of τ will depend on the details of the system.)

2. In the previous assignment, we (hopefully) found that the radiative "lifetime" of an atom in the classical oscillator model was

$$t_c = \frac{1}{\gamma_R} = \frac{6\pi\epsilon_0 mc^3}{\omega_0^2 e^2}.$$

In comparison, the quantum mechanical calculation of the lifetime of the 2P state of hydrogen gives

$$t_q = \frac{3^{11} m^2 c^3 e^2}{\pi 2^{19} \hbar^3 \omega_0^3 \epsilon_0},$$

which looks rather different. Show algebraically, however, that $t_c/t_q = 2^{13}/3^9 \approx 0.41$. (This will require an expression for the transition frequency ω_0.) Thus, the two models are in reasonable correspondence.

3. Sodium atoms have a doublet of excited states labelled $3\mathrm{p}_{1/2}$ and $3\mathrm{p}_{3/2}$ that are connected to the $3\mathrm{s}_{1/2}$ ground state by transitions at 589.6 nm and 589.0 nm repectively. The excited states have a radiative lifetime of 16 ns and degeneracies of 2 and 4, while the ground state has degeneracy 2. At a temperature of 400 K, Na vapor has a number density $N = 3 \times 10^{11}$ cm^{-3} and the collision rate between atoms is of order 1000 s^{-1}. Identify the dominant source of line broadening in this case, and then find the peak absorption coefficient α for each of the two lines. (Assume the incident light is weak enough to ignore saturation effects.)

4. What are the on-resonance saturation intensities for the two Na transitions of the previous problem?

822 students only:

5. Suppose a cell of length L contains a spin-polarized gas which is sufficiently cold and dilute that collisional and Doppler broadening are small compared to the radiative decay rate $1/t_s$. The cell is placed in a static magnetic field such that $B(z) = \beta z$, with $z = 0$ in the middle of the cell. The Zeeman effect will then shift the transition frequency in a position-dependent way, according to $\omega_0(z) = \omega_0(0) + hB(z)$ for some constant h. This gives rise to a simple form of inhomogeneous broadening.
(a) If the radiative linewidth is small compared to $h\beta L$, what is the lineshape function $g(\nu)$?
(b) Calculate $g(\nu)$ when the radiative and inhomogeneous linewidths are similar. Sketch the line shape for $h\beta L = 1/t_s$.