Phys 532 Assignment 7 Due March 21

1. The semiconductor GaAs has the following parameters:

$$E_{\text{gap}} = 1.424 \text{ eV}$$
$$m_c = 0.067\ m_{\text{electron}}$$
$$m_v = 0.46\ m_{\text{electron}}$$
$$x_{\text{vc}} = 3.2\ \text{Å}$$
$$n = 3.64$$

If the semiconductor is excited such that $N_c = N_v = 2 \times 10^{18}\ \text{cm}^{-3}$, evaluate and sketch a plot of the gain coefficient $\gamma(\nu)$ at zero temperature. In particular, indicate the (vacuum) wavelength at which gain occurs, the total frequency range across which the gain extends, and the peak gain achieved. (For comparison, Yariv Fig. 11.6b shows a similar calculation at room temperature.)

2. Show that when light of frequency ω is absorbed in a semiconductor, the crystal momentum of the excited electron is

$$k = \sqrt{\frac{2m_r}{\hbar^2}(\hbar\omega - E_{\text{gap}})},$$

and thus the energies of the conduction and valence states involved are

$$E_c = \frac{m_r}{m_c}(\hbar\omega - E_{\text{gap}})$$

and

$$E_v = \frac{m_r}{m_v}(\hbar\omega - E_{\text{gap}}).$$

Here m_r is the reduced mass $(m_v m_c)/(m_v + m_c)$.

3. The expressions we derived for the absorption coefficients of a semiconductor and a two-level molecule are

$$\alpha_{\rm sc} = \frac{\omega e^2 x_{vc}^2 (2m_r)^{3/2}}{2\pi\epsilon_0 n c\hbar^3}\sqrt{\hbar\omega - E_g}$$

and

$$\alpha_{\rm mol} = \frac{\lambda^2 N}{8\pi n^2 t_s} g(\nu),$$

which look rather different. They can be related by noting that the effective density of electrons for the semiconductor is

$$N_{\rm sc} \approx \rho(E_c)\Delta E_c$$

for c-band density of states $\rho(E_c)$ and energy range ΔE_c. Using the results of problem 2, E_c and ΔE_c can be related to the light frequency ω and the range of frequencies that those electrons can absorb, $\Delta\omega$. Also, the spontaneous emission time for a molecule is related to the dipole matrix element μ by

$$\frac{1}{t_s} = \frac{e^2\omega^3 n}{\pi\hbar c^3\epsilon_0}\mu^2.$$

Show that in both cases, the absorption coefficient is proportional to

$$\left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right)\frac{M^2\nu}{n\Delta\nu}N$$

where M is the appropriate dipole matrix element, ν the transition frequency, $\Delta\nu$ the appropriate frequency band width, and the term in parentheses can be recognized as the fine-structure constant. Determine the constants of proportionality in each case.

4. Suppose a GaAs diode laser is constructed with a rectangular gain region 100 nm high, 2 μm wide, and 500 μm long.
(a) Estimate the mode spacing for the cavity.
(b) Estimate the divergence angle of the output beam in the horizontal and vertical directions.

822 students only:
5. On pages 240-243, Yariv explains how to calculate the gain in a diode laser at non-zero temperature. The calculation is summarized graphically in Figure 11.5.
(a) Explain this figure by stating in words which curves correspond to which expressions and how the different curves are related.
(b) Redraw the figure for the zero-temperature case.