Phys 532 Assignment 8 Due March 28

1. In class, we considered a Pockel's cell constructed using an input polarizer with transmission axis at $+45°$ to vertical, an electro-optic crystal of length ℓ with axis vertical, and an output polarizer with transmission axis at $-45°$. (See also Yariv Fig. 14.7.) The index of refraction of the crystal is n_o for horizontally polarized light and $n_o + \Delta n$ for vertically polarized light. Thus the differential phase shift between the vertical and horizontal polarizations is $\phi = k\ell\Delta n$.
(a) Show, as claimed in class, that the transmitted intensity is $I_{\text{out}} = I_{\text{in}} \sin^2(\phi/2)$.
(b) Suppose that a quarter-wave plate with vertical axis is inserted before the final polarizer. Show that the intensity now responds linearly to a small applied shift Δn.

2. Consider a simple example of light propagating through a biaxial crystal, where $n_x \neq n_y \neq n_z$. Suppose the propagation direction of the light is $\hat{\mathbf{k}} = \sin\theta\hat{\mathbf{x}} + \cos\theta\hat{\mathbf{z}}$. Determine the directions and indices for the two principal polarizations of the light.

3. Consider a EO modulator constructed from $LiNbO_3$. Data for $LiNbO_3$ can be found in Yariv Table 14.2.
(a) Write out the equation for the index ellipsoid in the presence of an arbitrary electric field $\mathbf{E} = (E_x, E_y, E_z)$. Take the effects of crystal symmetry into account.
(b) Decide on an optimum crystal orientation, laser polarization, and applied field for amplitude modulation, and draw a sketch of your configuration.
(c) If the crystal is 1 mm by 1 mm thick and 2 cm long, calculate the applied voltage required to generate a π phase shift.

4. The electro-optic effect can also be used for frequency modulation. For example, suppose a $LiNbO_3$ crystal of length ℓ is oriented with a laser beam propagating along x and polarized along z. An oscillating electric field $E_1 \cos \Omega t$ is applied along the z direction. The electric field of the laser itself oscillates as $E_L = E_0 \exp i(\omega_0 t - kx)$.
(a) Show that after exiting the crystal, the laser field has a time dependence of $\exp i(\omega_0 t + \delta \cos \Omega t)$, and determine δ in terms of the parameters given and the $LiNbO_3$ data.
(b) If the "instantaneous" frequency of the laser is defined by

$$\frac{dE_L}{dt} = i\omega(t)E_L,$$

find the range of instantaneous frequencies sampled by the laser output.
(c) In the limit $\delta \ll 1$, show that the laser electric field can be expressed as a sum of three components oscillating at ω_0, $\omega_0 + \Omega$, and $\omega_0 - \Omega$, and find their relative amplitudes. Frequency modulation can thus be thought of either as a variation of the instantaneous frequency, or as the generation of additional frequency components at $\omega_0 \pm \Omega$. (This is a little counter-intuitive when δ is small, since the instantaneous frequency never actually equals $\omega_0 \pm \Omega$!)

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(d) If δ is not small, it turns out that the modulated optical field can be expressed as

$$E_L = E_0 \sum_n A_n(\delta) \exp i(\omega_0 t + n\Omega t).$$

Determine the coefficients A_n. How large must δ be to have equal amplitudes for the "carrier" at $n = 0$ and the first "sidebands" at $n = \pm 1$?