532 Assignment 11 Solutions

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1. In general, $P^{2\omega} = \alpha (P^{\omega})^2$

so here we have

$$\varepsilon_{cw} = \alpha P^{\omega}_{cw} \quad \text{for CW laser}$$

For pulsed laser, $P^{\omega}_{pk} = \frac{T}{\Delta t} P^{\omega}_{avg}$

(here T is supposed to be the period, not the rep rate!)

So $$P^{2\omega}_{pk} = \alpha (P^{\omega}_{avg})^2 \frac{T^2}{\Delta t^2}$$

and $$P^{2\omega}_{avg} = \frac{\Delta t}{T} P^{2\omega}_{pk} = \alpha (P^{\omega}_{avg})^2 \frac{T}{\Delta t}$$

Then $$\varepsilon_{pulsed} = \frac{P^{2\omega}_{avg}}{P^{\omega}_{avg}} = (\alpha P^{\omega}_{avg}) \frac{T}{\Delta t}$$

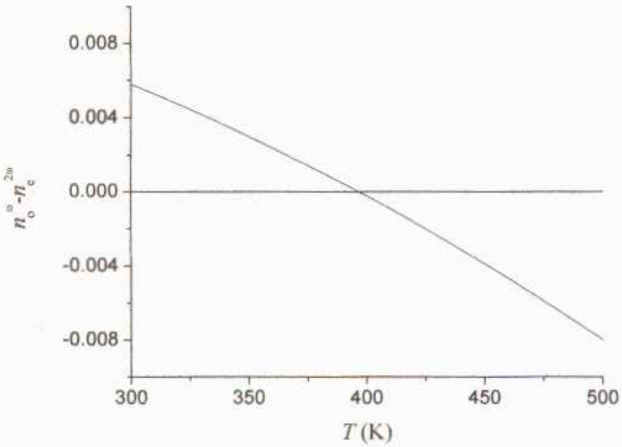
Since $P^{\omega}_{avg} = P^{\omega}_{cw}$ know

$$\alpha P^{\omega}_{avg} = \varepsilon$$

or $$\boxed{\varepsilon_{pulsed} = \varepsilon_{cw} \frac{T}{\Delta t} \gg \varepsilon_{cw}}$$

2. a) For NCPM, need $n_o(\omega) = n_e(2\omega)$

Easiest to just plot $n_o^{\omega} - n_e^{2\omega}$ vs T and find zero:



Get zero at $\boxed{T = 397\text{ K} = 124°\text{C}}$

b) Use $d_{31} = d_{zxx}$ nonlinear component
$= 4\times10^{-23}\ \frac{C}{V^2}$

Then $E^{2\omega} = -i\omega \frac{L}{n}\sqrt{\frac{\mu_0}{\varepsilon_0}}\, d_{31}\, (E_x^{\omega})^2$

(note, no factor of 2 because degenerate frequencies & polarizations)

Have $I^{\omega} = n\sqrt{\frac{\varepsilon_0}{\mu_0}}\ \frac{1}{2}\ |E^{\omega}|^2$

So $I^{2\omega} = n\ \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\ |E^{2\omega}|^2$

$$I^{2\omega} = \frac{n}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\ \omega^2\ \frac{L^2}{n^2}\ \frac{\mu_0}{\varepsilon_0}\ d_{31}^2\ |E^\omega|^4$$

$$= \frac{1}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\ \omega^2\ \frac{L^2}{n}\ d_{31}^2 \left(\frac{2}{n}\sqrt{\frac{\mu_0}{\varepsilon_0}}\ I^\omega\right)^2$$

$$= 2\left(\frac{\mu_0}{\varepsilon_0}\right)^{3/2} \frac{\omega^2 L^2}{n^3}\ d_{31}^2\ (I^\omega)^2$$

This is just spelling out stuff we also did in class

To get power, use $I^\omega(r) = I_0^\omega\ e^{-2r^2/w^2}$

Then $P^\omega = I_0^\omega \cdot \frac{\pi w^2}{2}$

or $I^\omega(r) = \frac{2P^\omega}{\pi w^2}\ e^{-2r^2/w^2}$

So,

$$I^{2\omega}(r) = \frac{8}{\pi^2}\left(\frac{\mu_0}{\varepsilon_0}\right)^{3/2} \frac{\omega^2 L^2}{n^3}\ d_{31}^2\ \frac{(P^\omega)^2}{w^4}\ e^{-4r^2/w^2}$$

and $P^{2\omega} = \int d^2r\ I^{2\omega}(r)$

$$= \frac{8}{\pi^2}\left(\frac{\mu_0}{\varepsilon_0}\right)^{3/2} \frac{\omega^2 L^2}{n^3}\ d_{31}^2\ \frac{(P^\omega)^2}{w^4}\ \frac{\pi w^2}{4}$$

$$= \frac{2}{\pi}\left(\frac{\mu_0}{\varepsilon_0}\right)^{3/2} \frac{\omega^2 L^2}{n^3 w^2}\ d_{31}^2\ (P^\omega)^2$$

Finally, optimum output if $\pi w^2 = \lambda z_0$

with $z_0 = \frac{L}{2}$

$\lambda = \frac{\lambda_0}{n} = \frac{2\pi c}{n\omega}$

Then $\omega^2 = \frac{1}{\pi} \frac{2\pi c}{n\omega} \frac{L}{2} = \frac{cL}{n\omega}$

and $\boxed{P^{2\omega} = \frac{2}{\pi} \left(\frac{\mu_0}{\varepsilon_0}\right)^{3/2} \frac{\omega^3 L}{n^2 c} d_{31}^2 (P^{\omega})^2}$

Factor of 4 different from result in class,
due to extra factor of 2 at begining.

Evaluate

$$P^{2\omega} = \frac{2}{\pi} (377\Omega)^3 \frac{(1.71\times10^{15}\,s^{-1})^3 (0.03m)}{(2.3)^2 (3\times10^8\,\frac{m}{s})} \left(4\times10^{-23} \frac{C}{V^2}\right)^2 (0.1W)^2$$

$$= 5.2\times10^{-5}\,W = \boxed{52\,\mu W}$$

3. a) Have $\frac{1}{\lambda_3} = \frac{1}{\lambda_1} - \frac{1}{\lambda_2}$

min: $\lambda_1 = 677\,nm$ $\lambda_2 = 811\,nm$

$\lambda_3 = 4.1\,\mu m$

max $\lambda_1 = 683\,nm$ $\lambda_2 = 805\,nm$

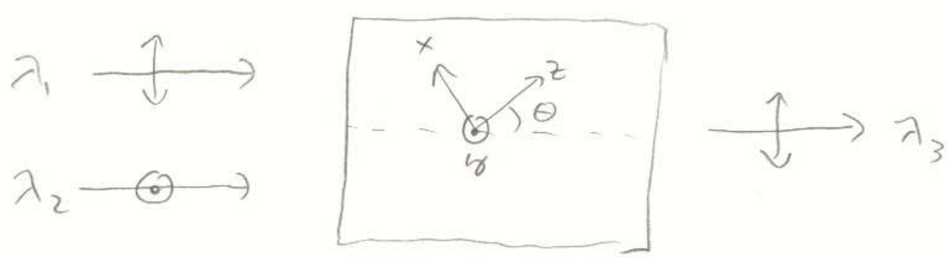
$\lambda_3 = 4.5\,\mu m$

λ_3 tunable from $\boxed{4.1 \text{ to } 4.5\,\mu m}$

b) Using $d_{16} = d_{xxy}$

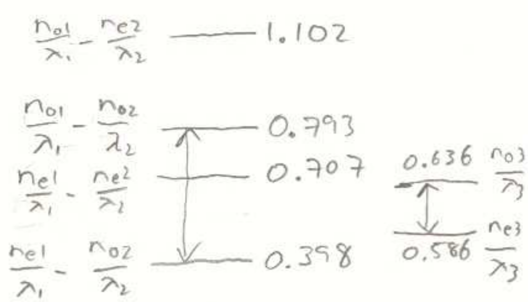
So need output with x component = extraordinary

So must have:



c) For phase matching, need

Diagram:



$$\frac{n_{e3}'}{\lambda_3} = \frac{n_{e1}'}{\lambda_1} - \frac{n_{e2}'}{\lambda_2}$$

$$\frac{1}{n_e'^2} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}$$

So,

$$f(\theta) = \frac{1}{\lambda_3}\left(\frac{\cos^2\theta}{n_{3o}^2} + \frac{\sin^2\theta}{n_{3e}^2}\right)^{-1/2} - \frac{1}{\lambda_1}\left(\frac{\cos^2\theta}{n_{1o}^2} + \frac{\sin^2\theta}{n_{1e}^2}\right)^{-1/2} + \frac{n_{2o}}{\lambda_2} = 0$$

Solve numerically, plot and find where $f(\theta) = 0$

Find $\boxed{\theta_m = 0.70 \text{ rad} = 40^\circ}$

Couple only x components at λ_1 and λ_3,
so get angle factor of $\cos^2\theta$

Thus $d' = d_{16} \cos^2\theta$

$= (-18 \frac{pm}{V}) \times 0.58$

$$\boxed{d' = -10.5 \frac{pm}{V}}$$

4. a) Have $\hat{k} = \sin\theta \, \hat{x} + \cos\theta \, \hat{z}$

$$\hat{S} = \frac{\vec{E} \times \vec{H}}{|\vec{E}||\vec{H}|}$$

Given $\vec{H} = H\hat{y}$

Also know $\vec{E} = E_x \hat{x} + E_z \hat{z}$

with

$$\frac{E_x}{E_z} = \frac{\sin\theta \, (n^2 - n_e^2)}{\cos\theta \, (n^2 - n_o^2)}$$

Then $\vec{E} \times \vec{H} = \begin{vmatrix} x & y & z \\ E_x & 0 & E_z \\ 0 & H & 0 \end{vmatrix}$

$$= \hat{x}(-E_z H) + \hat{z}(E_x H)$$

and
$$\hat{S} = \frac{-E_z \hat{x} + E_x \hat{z}}{\sqrt{E_x^2 + E_z^2}}$$

Then $\hat{S} \cdot \hat{k} = \dfrac{-E_z \sin\theta + E_x \cos\theta}{\sqrt{E_x^2 + E_z^2}}$

$$= \pm \frac{\left(\frac{E_x}{E_z}\right)\cos\theta - \sin\theta}{\sqrt{\left(\frac{E_x}{E_z}\right)^2 + 1}}$$

(Need − sign if $E_z < 0$)

$$\cos\beta = \pm \frac{\sin\theta \left(\frac{n^2 - n_e^2}{n^2 - n_o^2}\right) - \sin\theta}{\sqrt{\frac{\sin^2\theta}{\cos^2\theta}\left(\frac{n^2 - n_e^2}{n^2 - n_o^2}\right)^2 + 1}}$$

$$= \frac{\sin\theta\cos\theta\,(n_o^2 - n_e^2)}{\sqrt{\sin^2\theta\,(n^2 - n_e^2) + \cos^2\theta\,(n^2 - n_o^2)}}$$

Now use

$$\frac{n^2 - n_e^2}{n^2 - n_o^2} = \frac{1 - \frac{n_e^2}{n^2}}{1 - \frac{n_o^2}{n^2}}$$

$$= \frac{1 - n_e^2\left(\frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}\right)}{1 - n_o^2\left(\frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}\right)}$$

$$= \frac{1 - \sin^2\theta - \frac{n_e^2}{n_o^2}\cos^2\theta}{1 - \cos^2\theta - \frac{n_o^2}{n_e^2}\sin^2\theta}$$

$$= \frac{\cos^2\theta\left(1 - \frac{n_e^2}{n_o^2}\right)}{\sin^2\theta\left(1 - \frac{n_o^2}{n_e^2}\right)}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}\,\frac{n_e^2}{n_o^2}\,\frac{\left(\frac{1}{n_e^2} - \frac{1}{n_o^2}\right)}{\left(\frac{1}{n_o^2} - \frac{1}{n_e^2}\right)}$$

$$= -\frac{\cos^2\theta}{\sin^2\theta}\,\frac{n_e^2}{n_o^2}$$

So,
$$\cos\beta = \pm \frac{-\frac{\cos^2\theta}{\sin\theta}\frac{n_e^2}{n_o^2} - \sin\theta}{\sqrt{\frac{\cos^2\theta}{\sin^2\theta}\frac{n_e^4}{n_o^4} + 1}}$$

$$= \mp \frac{n_e^2\cos^2\theta + n_o^2\sin^2\theta}{\sqrt{n_e^4\cos^2\theta + n_o^4\sin^2\theta}}$$

To determine $\pm$, note for $\theta = 0$, should have no walk off: $\cos\beta \to 1$

So, need $+$ sign

$$\boxed{\cos\beta = \frac{n_e^2\cos^2\theta + n_o^2\sin^2\theta}{\sqrt{n_e^4\cos^2\theta + n_o^4\sin^2\theta}}}$$

b) $\sin^2\beta = 1 - \cos^2\beta$

$$= 1 - \frac{n_e^4\cos^4\theta + 2n_e^2n_o^2\sin^2\theta\cos^2\theta + n_o^4\sin^4\theta}{n_e^4\cos^2\theta + n_o^4\sin^2\theta}$$

$$= \frac{n_e^4\cos^2\theta(1-\cos^2\theta) - 2n_e^2n_o^2\sin^2\theta\cos^2\theta + n_o^4\sin^2\theta(1-\sin^2\theta)}{n_e^4\cos^2\theta + n_o^4\sin^2\theta}$$

$$\sin^2\beta = \frac{\sin^2\theta\cos^2\theta\,(n_e^4 - 2n_e^2 n_o^2 + n_o^4)}{n_e^4\cos^2\theta + n_o^4\sin^2\theta}$$

so

$$\boxed{\sin\beta = \frac{\sin\theta\cos\theta\,(n_e^2 - n_o^2)}{\sqrt{n_e^4\cos^2\theta + n_o^4\sin^2\theta}}}$$

c) For exemple, $n_o^{2\omega} = 1.5357$

$n_e^{2\omega} = 1.4897$

$\theta = 53.5°$

$|\sin\beta| = 0.0288$

$$\boxed{\beta = 1.7°}$$