Phys 532 Assigment 12 Solutions

1. Have parametric gain

$$g_0 = \cosh^2 \gamma L - 1 \approx \gamma^2 L^2 \quad \text{for small } g_0$$

$$= \varkappa^2 L^2 A_3^2$$

$$= \frac{\mu_0}{\varepsilon_0} \frac{\omega_1 \omega_2 \omega_3}{n_1 n_2 n_3} d'^2 L^2 \frac{n_3}{\omega_3} |E^{\omega_3}|^2$$

$$= 2 \left(\frac{\mu_0}{\varepsilon_0}\right)^{3/2} \frac{\omega_1 \omega_2}{n_1 n_2 n_3} d'^2 L^2 I^{\omega_3}$$

$$= 2 \left(\frac{\mu_0}{\varepsilon_0}\right)^{3/2} \frac{\omega_1 \omega_2}{n_1 n_2 n_3} d'^2 L^2 \frac{1}{\pi w_0^2} P^{\omega_3}$$

where w_0 = cavity beam waist

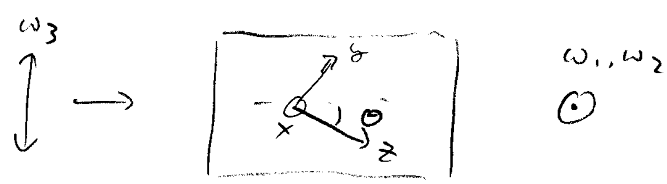
For optimum focusing, $\pi w_0^2 \approx \frac{\lambda_2 L}{2 n_2}$

Define $g_0 = \frac{P^{\omega_3}}{P_0}$

Then

$$P_0 = \frac{1}{4} \left(\frac{\varepsilon_0}{\mu_0}\right)^{3/2} \frac{\lambda_2}{\omega_1 \omega_2} \frac{n_1 n_3}{L d'^2}$$

Need to know d': set up is



So get $\cos\Theta$ from E_3 term

$$d' = d_{21} \cos\Theta = 1.6 \times 10^{-23} \frac{V}{c^2}$$

So,

$$P_0 = \frac{1}{4}\left(\frac{1}{377\Omega}\right)^3 \frac{(890nm)^2(590nm)}{(2\pi)^2(3\times10^8 m/s)^2} \frac{(1.6)^2}{(0.01m)(1.6\times10^{-23}\frac{V}{c^2})^2}$$

$$= 614\ W$$

Then threshold at $g_0 = L + T = 0.1$

so $\boxed{P_{3t} = 0.1\ P_0 = 61\ W}$

If $P_3 = 1000W$, use

$$P_{out} \approx \frac{T}{T+L_i} \frac{\omega_1}{\omega_3} (P_{pump} - P_{th})$$

$$= \frac{1}{2} \frac{355nm}{590nm} (1000 - 61)$$

$$\boxed{P_{out} = 280\ W}$$

2. (a)

$$\frac{d}{d\theta}(n_3\omega_3) = \omega_3 \frac{dn_3}{d\theta} \quad \text{since } \omega_3 \text{ is fixed}$$

also
$$\frac{d}{d\theta}(n_1\omega_1 + n_2\omega_2) = \omega_1 \frac{\partial n_1}{\partial \omega}\frac{d\omega_1}{d\theta} + n_1 \frac{d\omega_1}{d\theta}$$
$$\omega_2 \frac{\partial n_2}{\partial \omega}\frac{d\omega_2}{d\theta} + n_2 \frac{d\omega_2}{d\theta}$$

But also, $\omega_3 = \omega_1 + \omega_2$

so $\omega_2 = \omega_3 - \omega_1$

$$\frac{d\omega_2}{d\theta} = -\frac{d\omega_1}{d\theta} \quad \text{since } \omega_3 \text{ is fixed}$$

So, have

$$\boxed{\omega_3 \frac{dn_3}{d\theta} = \left[n_1 - n_2 + \omega_1 \frac{\partial n_1}{\partial \omega} - \omega_2 \frac{\partial n_2}{\partial \omega}\right]\frac{d\omega_1}{d\theta}}$$

b)
$$n_3(\theta) = \left(\frac{\cos^2\theta}{n_{3o}^2} + \frac{\sin^2\theta}{n_{3e}^2}\right)^{-1/2}$$

$$\frac{dn_3}{d\theta} = -\frac{1}{2}\left(-\frac{2\sin\theta\cos\theta}{n_{3o}^2} + \frac{2\sin\theta\cos\theta}{n_{3e}^2}\right)\left(\frac{\cos^2\theta}{n_{3o}^2} + \frac{\sin^2\theta}{n_{3e}^2}\right)^{-3/2}$$

$$\boxed{\frac{dn_3}{d\theta} = \frac{n_3^3}{2}\left(\frac{1}{n_{3o}^2} - \frac{1}{n_{3e}^2}\right)\sin 2\theta}$$

2.(c) So,

$$\frac{d\omega_1}{d\theta} = \frac{\omega_3 \frac{dn_3}{d\theta}}{n_1 - n_2 + \omega_1 \frac{\partial n_1}{\partial \omega} - \omega_2 \frac{\partial n_2}{\partial \omega}}$$

$$\boxed{\frac{d\omega_1}{d\theta} = \omega_3 \frac{n_3^3}{2} \sin 2\theta \frac{\frac{1}{n_{3o}^2} - \frac{1}{n_{3e}^2}}{n_1 - n_2 + \omega_1 \frac{\partial n_1}{\partial \omega} - \omega_2 \frac{\partial n_2}{\partial \omega}}}$$

(d) Since $\frac{1}{\omega}\frac{d\omega}{d\theta} = -\frac{1}{\lambda}\frac{d\lambda}{d\theta}$

and $\omega \frac{\partial n}{\partial \omega} = -\lambda \frac{\partial n}{\partial \lambda}$, here

from $\frac{d\omega}{d\lambda} = -\frac{\omega}{\lambda}$

$$-\frac{1}{\lambda_1}\frac{d\lambda_1}{d\theta} = \frac{\omega_3}{\omega_1} \frac{n_3^3}{2} \sin 2\theta \frac{\frac{1}{n_{3o}^2} - \frac{1}{n_{3e}^2}}{n_1 - n_2 + \lambda_2 \frac{\partial n_2}{\partial \lambda} - \lambda_1 \frac{\partial n_1}{\partial \lambda}}$$

Use $\lambda_1 = 590\,nm$

$\lambda_2 = 890\,nm$

$\lambda_3 = 355\,nm$

$n_1 - n_2 = 0.01$

$$\lambda_2 \frac{\partial n_2}{\partial \lambda} - \lambda_1 \frac{\partial n_1}{\partial \lambda} \approx (3.3 \times 10^{-5}\,nm^{-1})(890 - 590\,nm) = 0.01$$

$$\frac{1}{n_{3o}^2} - \frac{1}{n_{3e}^2} = \frac{1}{1.72^2} - \frac{1}{1.586^2} = -0.06$$

and $n_3 \approx n_1 \approx n_2 = 1.66$

Then $\frac{d\lambda_1}{d\theta} = - \frac{(590\,nm)^2}{355\,nm} \frac{(1.66)^3}{2} \frac{(-0.06)}{0.01+0.01} \sin 74°$

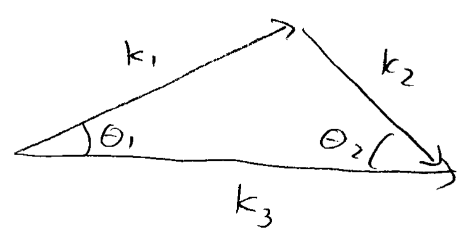
$= 6700 \frac{nm}{rad}$

$= 120 \frac{nm}{degree}$

So, if we want $\Delta\lambda_1 = 10\,nm$, need

$$\boxed{\begin{aligned}\Delta\theta \simeq \frac{10}{120} &= 0.085 \text{ degrees} \\ &= 1.5 \text{ mrad}\end{aligned}}$$

3. For phase matching, need $\vec{k}_3 = \vec{k}_1 + \vec{k}_2$



Also, $LiNbO_3$ has $d_{31}, d_{32}, d_{21}, d_{22}, d_{16}$,
d_{24} and d_{15} $\neq 0$

Since ω_3 polarization is along z, need to use $d_{31} = d_{zxx}$
or $d_{32} = d_{zyy}$

d_{zxx} would give o-polarized output
d_{zyy} gives e-polarized output

Know we need output $\perp$ to input for phase matching, so both output beams must be polarized $\boxed{\text{along } x}$

Therefore have $n_1 = n_{o1}$ and $n_2 = n_{o2}$.

So, $k_1 = \dfrac{2\pi n_{o1}}{\lambda_1}$ $\quad k_2 = \dfrac{2\pi n_{o2}}{\lambda_2}$

$$k_3 = \frac{2\pi n_{e3}}{\lambda_3}$$

Have $\lambda_1 = 2\,\mu m$, $\lambda_3 = 532\,nm$

also, $\omega_3 = \omega_1 + \omega_2$

so $\dfrac{1}{\lambda_3} = \dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}$

and $\lambda_2 = 725\,nm$

Then

$n_o(2\,\mu m) = 2.1981$

$n_o(725\,nm) = 2.2685$

$n_e(532\,nm) = 2.233$

From phase matching triangle, have

$$\cos\theta_1 = \frac{k_1^2 + k_3^2 - k_2^2}{2 k_1 k_3}$$

$$= \frac{\frac{n_{o1}^2}{\lambda_1^2} + \frac{n_{e3}^2}{\lambda_3^2} - \frac{n_{o2}^2}{\lambda_2^2}}{2 \frac{n_{o1}}{\lambda_1} \frac{n_{e3}}{\lambda_3}}$$

$$= 0.979$$

so $\boxed{\theta_1 = 11.7^\circ}$