Phys 532 Assignment 5 Solutions

1. Rate equations are

$$\frac{dN_1}{dt} = - - R N_1 + \Gamma_2 N_2 + R N_3 \qquad (1)$$

$$\frac{dN_2}{dt} = + \Gamma_3 N_3 - \Gamma_2 N_2 \qquad (2)$$

$$\frac{dN_3}{dt} = + R N_1 - \Gamma_3 N_3 - R N_3 \qquad (3)$$

Solve in steady state $\frac{d}{dt} \to 0$

from (2), $N_3 = \frac{\Gamma_2}{\Gamma_3} N_2$

from (3), $R N_1 - (\Gamma_3 + R) \frac{\Gamma_2}{\Gamma_3} N_2 = 0$

So $N_2 = \frac{\Gamma_3}{\Gamma_2} \frac{R}{\Gamma_3 + R} N_1$

Get inversion if $N_2 > N_1$, so need

$$\frac{\Gamma_3}{\Gamma_2} \frac{R}{\Gamma_3 + R} > 1$$

$$\Gamma_3 R > \Gamma_2 \Gamma_3 + \Gamma_2 R$$

$$(\Gamma_3 - \Gamma_2) R > \Gamma_2 \Gamma_3$$

or

$$\boxed{R > \frac{\Gamma_2 \Gamma_3}{\Gamma_3 - \Gamma_2}}$$

2. Gain coefficient is

$$\gamma = -\frac{1}{8\pi} \lambda_{12}^2 \Delta N \frac{g(\nu)}{t_s}$$

here $g(\nu) \approx \frac{1}{\Delta\nu_{12}} = \frac{1}{10\,GHz}$

Need ΔN: from rate equations in class,

$$-\Delta N = RN\left(\frac{1}{\Gamma_2} - \frac{g_2}{g_1}\frac{1}{\Gamma_1}\right)$$

where R is pump rate on $0 \to 3$ transition,
and we assume $R \ll \Gamma$'s

For optical pumping, have

$$R = \frac{1}{8\pi} \lambda_{03}^2 \frac{I}{h\nu_{03}} \frac{g_3}{g_0} \frac{g(\nu)}{t_s}$$

with $g(\nu) \approx \frac{1}{\Delta\nu_{03}} = \frac{1}{100\,GHz}$

$= 1140\ s^{-1}$ which is indeed small compared to Γ's.

So, $-\Delta N = (1140\,s^{-1})(30\,ns - \frac{1}{2}5\,ns)\,N$

$= 3.1 \times 10^{-5}\ N$

$= 3.1 \times 10^{18}\ m^{-3}$

and $\gamma = \frac{1}{8\pi}\left(\frac{800\,nm}{1.5}\right)^2 (3.1 \times 10^{18}\,m^{-3}) \frac{1}{10\,GHz \cdot 30\,ns}$

$= 117\ m^{-1} = \boxed{1.2\ cm^{-1}}$

3. Total loss per pass is $1\% + \frac{1}{2} \times 4\% = 3\%$

available gain is $0.2\,m^{-1} \times 20\,cm = 0.04$,

so laser will oscillate

In steady state,

$$\gamma = \frac{\gamma_0}{1 + I/I_s} = 0.03$$

so $$1 + \frac{I}{I_s} = \frac{4}{3}$$

$$I = \frac{1}{3} I_s$$

have $I_{sat} = \dfrac{4\pi h\nu}{\lambda^2 g(\nu)}$ at peak, $g(\nu) = 2T_2 = 4t_s$

$$I_{sat} = \frac{\pi h \nu}{\lambda^2 t_s}$$

$$= 229 \frac{W}{m^2}$$

So $I_{cav} = 76 \frac{W}{m^2}$ = I in cavity

and $I_{out} = T\, I_{cav} = 0.04 \times I_{cav}$

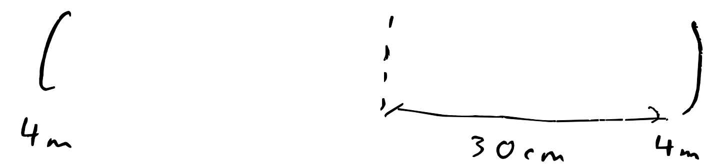
$$= 3.5 \frac{W}{m^2}$$

To get power, need beam area.

Analyze cavity:

3. (cont)

This looks like half of symmetric cavity



From Yeriv 7.1-9, $w_{mirror} = \sqrt{\frac{\lambda \ell}{2\pi}} \left[\frac{2R^2}{\ell(R-\frac{\ell}{2})}\right]^{1/4}$

for $\ell = 60\,cm$

and $w_0 = \sqrt{\frac{\lambda}{\pi}} \left[\frac{\ell}{2}\left(R-\frac{\ell}{2}\right)\right]^{1/4}$

$w_{mirror} = 370\,\mu m$

$w_0 = 342\,\mu m$

So, beam waist is reasonably uniform $w \approx 350\,\mu m$

Then $P \approx \pi w^2 I$

$$\boxed{P = 1.3\,\mu W}$$

For optimum coupling, $T = -L_i + \sqrt{g_0 L_i}$ (L_i and g_0 per round trip)

$= -0.02 + \sqrt{.02 \times .08} = \boxed{0.02}$

Then $P_{out} = I_s A\left(\sqrt{g_0} - \sqrt{L}\right)^2$ (Yeriv 9.3-18)

$= 230\,\frac{W}{m^2} \times \pi w^2 \left(\sqrt{.08} - \sqrt{.02}\right)^2 = \boxed{1.8\,\mu W}$

4. Have $\nu_m' = \dfrac{\nu_m}{1 + \chi'/2}$

where ν_m' = mode frequency with gas present

ν_m = mode frequency w/o gas

Here $\chi' = \dfrac{2(\nu_0 - \nu)}{\Delta\nu} \chi''$

what is χ''?

Know $\alpha_0 = k\, \chi''(\nu_0)$

and $\chi''(\nu) = \chi''(\nu_0) \dfrac{g(\nu)}{g(\nu_0)}$

with $g(\nu) = \dfrac{2T_2}{1 + 4\pi^2 T_2^2 (\nu - \nu_0)^2}$

$\Delta\nu = \dfrac{1}{\pi T_2}$

thus $g(\nu) = \dfrac{2}{\pi \Delta\nu} \dfrac{1}{1 + 4\dfrac{(\nu - \nu_0)^2}{\Delta\nu^2}}$

and $g(\nu_0) = \dfrac{2}{\pi \Delta\nu}$

Putting together,

$$\chi' = \frac{2(\nu_0 - \nu)}{\Delta\nu} \frac{\alpha_0}{K} \frac{1}{1 + 4\frac{(\nu - \nu_0)^2}{\Delta\nu^2}}$$

4. (cont)

We're interested in ν_m' when $\nu_m = \nu_0 \pm \Delta\nu_L$

Take + case:

$$\nu_+' = \frac{\nu_0 + \frac{c}{2\ell}}{1 + \frac{1}{2}\chi'(\nu_+')} = \nu_0 + \delta$$

$$= \frac{\nu_0 + \frac{c}{2\ell}}{1 + \frac{(-\delta)}{\Delta\nu}\frac{\alpha_0}{k}\left(\frac{1}{1+4\frac{\delta^2}{\Delta\nu^2}}\right)}$$

But we know $\Delta\nu \gg \frac{c}{2\ell}$, so expect $\Delta\nu \gg \delta$

So, $$\nu_+' \simeq \frac{\nu_0 + \frac{c}{2\ell}}{1 - \frac{\delta}{\Delta\nu}\frac{\alpha_0}{k}} = \nu_0 + \delta$$

$$\nu_0 + \frac{c}{2\ell} = (\nu_0 + \delta)\left(1 - \frac{\delta}{\Delta\nu}\frac{\alpha_0}{k}\right)$$

$$\frac{c}{2\ell} = \delta - \frac{\delta\nu_0}{\Delta\nu}\frac{\alpha_0}{k}$$

$$= \delta\left(1 - \frac{\nu_0\alpha_0}{\Delta\nu k}\right)$$

So $$\delta = \frac{c}{2\ell}\left(1 + \frac{\nu_0\alpha_0}{\Delta\nu k}\right)$$

note $k = \frac{\omega}{c} = \frac{2\pi\nu_0}{c}$

4. (cont)

$$\boxed{\delta = \Delta\nu_L \left(1 + \frac{\alpha_0 c}{2\pi\,\Delta\nu}\right)}$$

Check ... is $\frac{\alpha_0 c}{2\pi\Delta\nu} \ll 1$?

$$= \frac{\Delta\nu_L}{\Delta\nu} \frac{\alpha_0 c}{2\pi\,\Delta\nu_L}$$

$$= \frac{\Delta\nu_L}{\Delta\nu} \frac{\alpha_0 c}{2\pi \frac{c}{2\ell}}$$

$$= \underbrace{\frac{\Delta\nu_L}{\Delta\nu}}_{\ll 1} \underbrace{\frac{1}{\pi}\alpha_0 \ell}_{\not\ll 1}$$

So, shift is small, as we assumed.

Note shift is <u>away</u> from ν_0 ... looks like "frequency pushing" here

(difference is gain vs. absorption)