Phys 532 Assignment 6 Solutions

1. Doppler profile is $g(\nu) = \left(\frac{M\lambda^2}{2\pi kT}\right)^{1/2} e^{-\frac{M\lambda^2}{2kT}(\nu-\nu_0)^2}$

So if gain at peak of profile is $g_0 = 4\%$,

have

$$g_0(\nu) = g_0 e^{-\frac{M\lambda^2}{2kT}(\nu-\nu_0)^2}$$

To get lasing oscillation, need $g_0(\nu) > L = 1\%$

or,

$$e^{-\frac{M\lambda^2}{2kT}(\nu-\nu_0)^2} > \frac{1}{4}$$

solve for range of ν allowed:

$$(\nu-\nu_0)^2 < \frac{2kT}{M\lambda^2} \ln 4$$

$$= \frac{2\times 1.38\times 10^{-23}\frac{J}{K}\cdot 300K}{20\cdot 1.67\times 10^{-27}kg\cdot 633nm^2} \ln 4$$

(mass of neon is 20u)

$$(\nu-\nu_0)^2 < 8.58\times 10^{17}\ s^{-2}$$

$$|\nu-\nu_0| < 926\ MHz$$

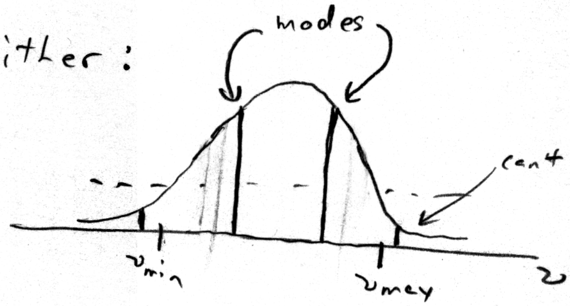
So, $\nu_{max} - \nu_{min} = 1.85\ GHz$

1. (cont)

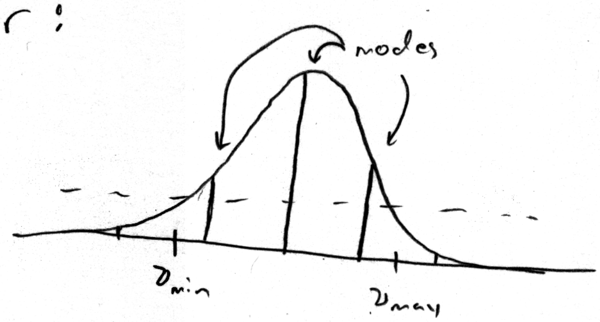
But longitudinal mode spacing is $\frac{c}{2\ell} = 500\,MHz$

There will then be $\boxed{2 \text{ or } 3 \text{ modes}}$ oscillating:

either:



or:



2. Know $\gamma = \frac{1}{8\pi} \lambda^2 \Delta N \frac{g(\nu)}{t_s} \approx \frac{1}{8\pi} \lambda^2 \Delta N \frac{1}{t_s \Delta\nu}$

where $\lambda = \frac{670\,nm}{n} = 504\,nm$

Need $\Delta N = NR\left(\frac{1}{\Gamma_2} - \frac{1}{\Gamma_1}\right)$

$N = 10^{24}\,m^{-3}$

$\frac{1}{\Gamma_2} = t_s = 20\,ns$

$\frac{1}{\Gamma_1} = 0.1\,ns$

2. (cont)

Need $R = W_{0\to3} = \frac{I}{h\nu_{03}} \frac{\alpha_{03}}{N}$

So,

$$\gamma_0 = \frac{1}{8\pi} \lambda^2 \frac{I}{h\nu_{03}} \alpha_{03} \left(\frac{1}{\Gamma_2} - \frac{1}{\Gamma_1}\right) \frac{1}{t_s \Delta\nu}$$

$$\simeq \frac{1}{8\pi} \lambda^2 \frac{\alpha_{03}}{h\nu_{03}} \frac{1}{\Delta\nu} I$$

$$= \frac{1}{8\pi} (504\,nm)^2 \cdot \frac{100\,m^{-1}}{6.626\times10^{-34}\,J\cdot s} \frac{514\,nm}{3\times10^8\,m/s} \frac{1}{1\,GHz} I$$

$$= 2.6\times10^{-3} \frac{m}{W} \cdot I .$$

Want $e^{\gamma_0 l} = 10$, so $\gamma_0 = 230\,m^{-1}$

which requires $\boxed{\begin{aligned} I &= 88000\ W/m^2 \\ &= 8.8\ W/cm^2 \end{aligned}}$

3. In propagating through thickness dz, intensity changes by

$$\frac{dI}{dz} = \gamma I$$

where $\gamma = \frac{\gamma_0}{1 + I/I_{sat}}$

So,

$$\frac{dI}{dz} = \frac{\gamma_0}{1 + I/I_s} I$$

$$\left(\frac{1}{I} + \frac{1}{I_s}\right) dI = \gamma_0 \, dz$$

or

$$\int_{I_{in}}^{I_{out}} \frac{1}{I} + \frac{1}{I_s} \, dI = \int_0^{l} \gamma_0 \, dz$$

for incident I_{in} + output I_{out}

$$\ln \frac{I_{out}}{I_{in}} + \frac{I_{out} - I_{in}}{I_s} = \gamma_0 l$$

here $I_{in} = 0.2 \, I_s$

$I_{out} = 2 \, I_s$, so

$$\gamma_0 = \frac{1}{l} \left(\ln 10 + 1.8 \right)$$

$$= 410 \text{ m}^{-1}$$

From previous problem,

$$\gamma_0 = 2.6 \times 10^{-3} \frac{m}{W} \cdot I, \text{ so}$$

$$\boxed{I_{pump} = 15.7 \, W/cm^2}$$