Phys 532 Assigment 7 Solutions

1. Have $\gamma = \dfrac{\omega e^2 x_{vc}^2 (2m_r)^{3/2}}{2\pi\varepsilon_0 n c \hbar^3}\sqrt{\hbar\omega - E_s}$

$= K\sqrt{\hbar\omega - E_s}$

for $E_{gap} < \hbar\omega < E_{gap} + \mathcal{E}_v + \mathcal{E}_c$

here $\mathcal{E}_v = \dfrac{\hbar^2}{2m_v}(3\pi^2 N_v)^{2/3} = 2.0\times10^{-21}\ \text{J}$

$= 0.0126\ \text{eV}$

$\mathcal{E}_c = \dfrac{m_v}{m_c}\mathcal{E}_v = 0.086\ \text{eV}$

So need $E_s < \hbar\omega < E_s + 0.1\ \text{eV}$, or $\omega \approx \dfrac{E_s}{\hbar}$

Then $\omega = \dfrac{2.28\times10^{-19}\ \text{J}}{1.054\times10^{-34}\ \text{J}\cdot\text{s}} = 2.16\times10^{15}\ \text{rad/s}$

and $\lambda = \dfrac{2\pi c}{\omega} \approx 870\ \text{nm}$

Also, $m_r = \dfrac{m_c m_v}{m_c + m_v} = \dfrac{0.067\cdot 0.46}{0.067 + 0.46} m_e = 0.059 m_e$

$= 5.33\times10^{-32}\ \text{kg}$

So,

$K = \dfrac{(2.16\times10^{15}\ \text{rad/s})(1.6\times10^{-19}\ \text{C})^2(3.2\times10^{-10}\ \text{m})^2(2\cdot5.33\times10^{-32}\ \text{kg})^{3/2}}{6.28\times 8.8\times10^{-12}\ \text{F/m}\times 3.64\times 3\times10^{8}\ \frac{\text{m}}{\text{s}}\times(1.054\times10^{-34}\ \text{J}\cdot\text{s})^3}$

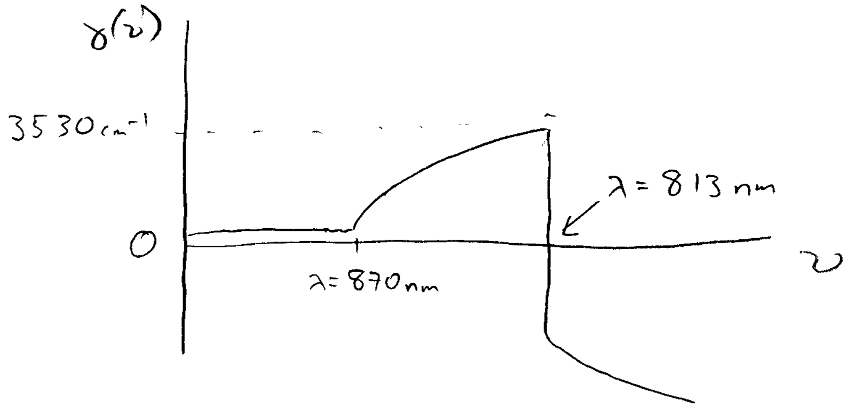
$= 2.79\times10^{15}\ \dfrac{\text{m}^{-1}}{\sqrt{\text{J}}} = 1.12\times10^{6}\ \dfrac{\text{m}^{-1}}{\sqrt{\text{eV}}}$

So at $\hbar\omega = E_g + \mathcal{E}_v + \mathcal{E}_c$, have

$$\sqrt{\hbar\omega - E_g} = \sqrt{0.1\,eV}$$

and $\gamma = 3530\ cm^{-1}$

So, $\gamma(\nu)$ looks like:



minimum λ for gain is at $\hbar\omega = 1.524\,eV$

$\lambda = 813\,nm$

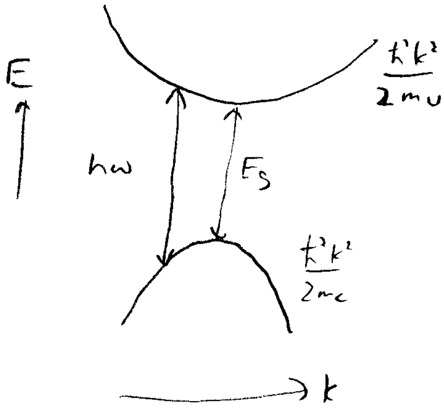
or, total frequency range is $\Delta\nu = \frac{c}{813\,nm} - \frac{c}{870\,nm}$

$= 2.4 \times 10^{13}\ Hz$

$= 810\ cm^{-1}$

2. By energy conservation, have

$$\hbar\omega = E_g + \frac{\hbar^2 k^2}{2m_c} + \frac{\hbar^2 k^2}{2m_v}$$



So $\hbar\omega = \frac{\hbar^2 k^2}{2m_r} + E_g$

for $\frac{1}{m_r} = \frac{1}{m_c} + \frac{1}{m_v}$

or $m_r = \frac{m_c m_v}{m_c + m_v}$

So, $k^2 = \frac{2m_r}{\hbar^2}(\hbar\omega - E_g)$

$$\boxed{k = \sqrt{\frac{2m_r}{\hbar^2}(\hbar\omega - E_g)}}$$

Then energy of conduction e^- is $\frac{\hbar^2 k^2}{2m_c} = \frac{m_r}{m_c}(\hbar\omega - E_g)$

and valence e^- is $\frac{\hbar^2 k^2}{2m_v} = \frac{m_r}{m_v}(\hbar\omega - E_g)$

3.

Have $N_{sc} = \rho(E_c)\,\Delta E_c$

where $\rho(E_c) = \frac{1}{2\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2} E_c^{1/2}$

From problem 2,

$$E_c = \frac{m_r}{m_c}(\hbar\omega - E_g)$$

and $\Delta E_c = \frac{m_r}{m_c}\hbar\,\Delta\omega$

So, we see $N = \frac{1}{2\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2}\sqrt{\frac{m_r}{m_c}(\hbar\omega - E_g)}\;\frac{m_r}{m_c}\hbar\,\Delta\omega$

$$= \frac{1}{2\pi^2}\frac{(2m_r)^{3/2}}{\hbar^2}\sqrt{\hbar\omega - E_g}\;\Delta\omega$$

So, in formula for α_{sc}, can replace

$\sqrt{\hbar\omega - E_g}$ by $2\pi^2\frac{\hbar^2}{(2m_r)^{3/2}}\frac{N}{\Delta\omega}$

to get

$$\alpha_{sc} = \frac{\pi\omega e^2 X_{vc}^2}{\varepsilon_0 n c\hbar}\frac{N}{\Delta\omega}$$

$$= 4\pi^2\left(\frac{e^2}{4\pi\varepsilon_0\hbar c}\right)\frac{X_{vc}^2\,\omega}{n\,\Delta\omega}N$$

$$\boxed{\alpha_{sc} = 4\pi^2\alpha_{fs}\frac{M^2\,\nu}{n\,\Delta\nu}N}$$

with $M^2 = X_{vc}^2$

$\frac{\nu}{\Delta\nu} = \frac{\omega}{\Delta\omega}$

While for molecules, have

$$\alpha_m = \frac{\lambda^2 N}{8\pi n^2} \frac{1}{\Delta\nu} \frac{e^2 \omega^3 n \mu^2}{\pi \hbar c^3 \varepsilon_0}$$

where $g(\nu) \approx \frac{1}{\Delta\nu}$

Then use $\omega = 2\pi\nu = \frac{2\pi c}{\lambda}$

$$\alpha_m = \frac{\lambda^2 N}{8\pi^2} \frac{e^2}{\hbar c \varepsilon_0} \frac{(2\pi)^3 \nu c^2}{n c^2 \lambda^2} \frac{\mu^2}{\Delta\nu}$$

$$= \pi \frac{e^2}{\varepsilon_0 \hbar c} \frac{\mu^2 \nu}{n \Delta\nu} N$$

$$\boxed{\alpha_m = 4\pi^2 \alpha_{fs} \frac{M^2 \nu}{n \Delta\nu} N}$$

Just like α_{sc}!

(Note, if you take $g(\nu) = \frac{2}{\pi\Delta\nu}$, get coefficient of 8π ... pretty close)

(6)

4. a) Expect $\Delta\nu_L \simeq \frac{c}{2nL}$, although not strictly accurate for such a small cavity

So $\boxed{\Delta\nu_L \approx 82 \text{ GHz}}$

b) Expect diffraction angle $\Delta\Theta \sim \frac{\lambda}{d}$

In horizontal direction,

$$\Delta\Theta \sim \frac{850\text{nm}}{2\,\mu\text{m}} \approx 0.4\text{rad}$$

$$\approx 25^\circ$$

In vertical direction, $\Delta\Theta \sim \frac{850\text{nm}}{100\text{nm}} \gg 1$

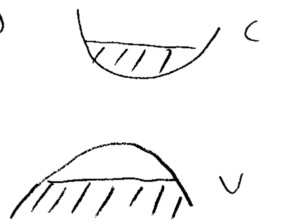
So, expect diffraction into nearly 180°

(Note that our diffraction approximations break down for such a small aperture.)

5. a)

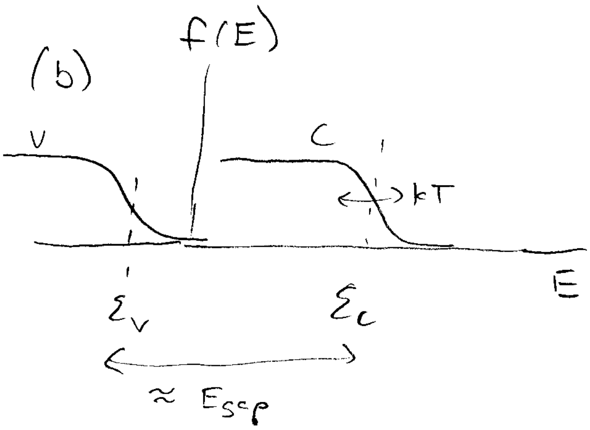
Four parts to figure:

(a)



This is figure we used in class, shows energy vs k for C and V bands

(b)



Shows the $T > 0$ Fermi-Dirac distributions for the electrons in the C and V bands

Here the quasi-Fermi levels are the energies where $f(E) = \frac{1}{2}$.

Distributions are rounded of by $\sim kT$

In Yariv figure, plotted sideways to line up with (a)

(c) For given ω, know E_v and E_c from problem 2.

Gain or absorption depends on population difference between ground and excited states

So, here plot $f_c(E_c) - f_v(E_v)$ as function of ω. Yariv calls $E_c = E_b$ and $E_v = E_a$, that is, E_a & E_b are energies of two states involved in the transition.

See that $f_c - f_v = 0$ at frequency where

$$E_c = \mathcal{E}_c \quad \text{and} \quad E_v = \mathcal{E}_c,$$

$$\text{or} \quad \hbar\omega = E_g + \mathcal{E}_v + \mathcal{E}_c$$

(d) Know that $\gamma \propto \sqrt{\hbar\omega - E_g}$, where square root comes from density of states ρ

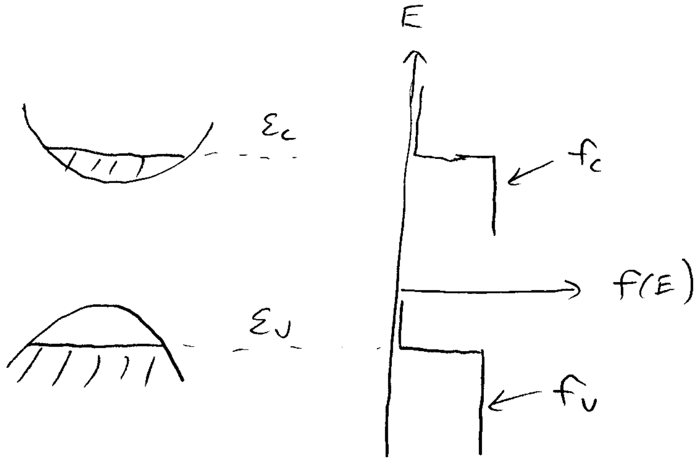
[Note that in problem 3, we saw how we can define a $\rho(\omega)$ by

$$\rho(\omega)\,d\omega = \rho(E)\,dE \;]$$

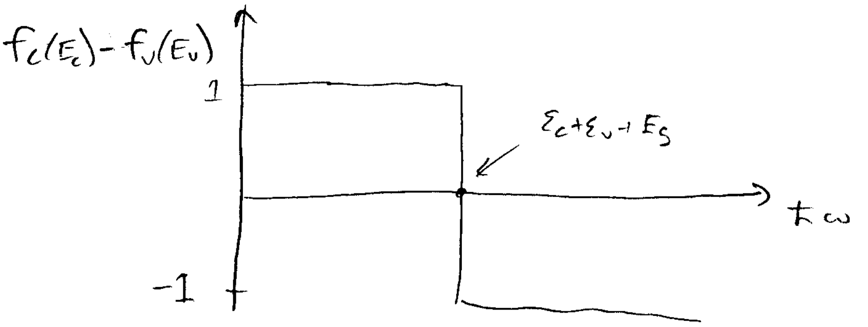
We expect to multiply this by the actual inversion, $f_c(\omega) - f_v(\omega)$

So, we multiply the curve from (c) by $\sqrt{\hbar\omega - E_g}$ to get actual gain/absorption curve

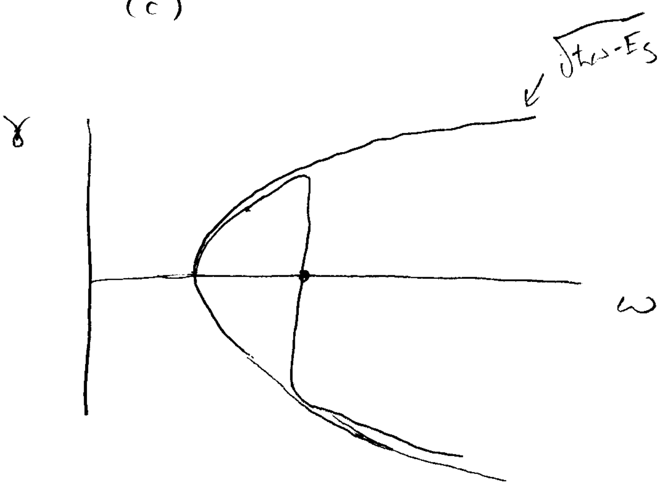
4. (b) At zero T, have:



(a) (b)



(c)



like from class

(d)