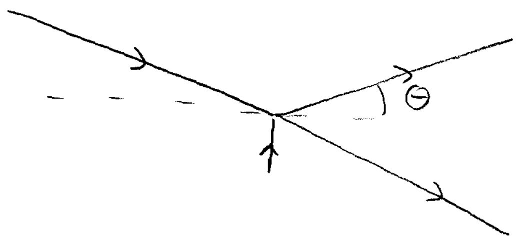
Phys 532 Assignment 9 Solutions

1. a) We found $\Theta = \frac{k_s}{2k} = \frac{\lambda}{2n}\frac{\nu_s}{v_s} = \frac{780\,nm}{2\cdot 2.35} \cdot \frac{80\,MHz}{617\,m/s} = 0.022$



Angle between beams is $\boxed{2\Theta = 0.043\,rad = 2.5^\circ}$

b) Time for sound wave to cross beam is

$$T \approx \frac{w}{v_s} = \frac{200\,\mu m}{617\,m/s} = 325\,ns$$

For modulation frequency f, need to turn light on, off, and on again in time $\frac{1}{f}$

So, max $f \approx \frac{1}{2T} \approx \boxed{1.5\,MHz}$

2. Peak power $= \frac{\text{energy}}{\text{duration}} = \frac{1\,J}{5\,ns} = \boxed{2\times10^{8}\,W}$

Intensity $I \approx \frac{P}{\pi w^2} = \frac{2\times10^{8}\,W}{\pi\cdot(50\,\mu m)^2} = \boxed{2.5\times10^{16}\,\frac{W}{m^2}}$

Electric Field: $I = \frac{1}{2}\varepsilon_0 c|E|^2$ so

$$E = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2\times 2.5\times10^{16}\,W/m^2}{8.8\times10^{-12}\,\frac{J}{V^2 m}\cdot 3\times10^{8}\,\frac{m}{s}}}$$

$$\boxed{E = 4.3\times10^{9}\,\frac{V}{m}}$$

2. (cont)

Average power is energy × (rep rate)

$$\boxed{P_{avg} = 1\,W}$$

3. a) Need to know inversion ΔN

Know energy of excited ions is 0.2 J

Each excited ion has energy $h\nu = \frac{hc}{\lambda}$

$$= 1.9 \times 10^{-19}\,J$$

So, number of excited ions is

$$n_i = \frac{0.2\,J}{1.9\times10^{-19}\,J} = 1.1\times10^{18}$$

Then density $\Delta N = \frac{n}{\text{volume}} = \frac{1.1\times10^{18}}{10\,cm^3} = 1.1\times10^{17}\,cm^{-3}$

So, $\gamma_0 = \frac{(1.064\,\mu m)^2 (1.1\times10^{17}\,cm^{-3})}{8\pi (1.5)^2 (10^{-3}s)(2\times10^{11}\,Hz)} = 10.7\,m^{-1}$

and $g_0 = e^{2\gamma_0 L} = e^{2.1} = \boxed{8.5 \gg \Gamma = 0.3}$

b) We found that for $g_0 \gg \Gamma$, expect

$$P_{out} = \frac{h\nu}{t_c}\frac{n_i}{2}$$

3. (cont)

Here $t_c = \frac{1}{\Gamma \Delta \nu_L}$ $\qquad \Delta \nu_L = \frac{c}{2 l_{opt}}$

$l_{opt} = 20\,cm + 1.5 \times 10\,cm = 35\,cm$

$$t_c = \frac{70\,cm}{0.3 \cdot 3\times10^8\,m/s} = 7.8\,ns$$

$$P_{max} = \frac{(6.63\times10^{-34}\,J\,s)(3\times10^8\,m/s)}{(1.064\,\mu m)(7.8\,ns)} \quad \frac{1.1\times10^{18}}{2}$$

$$\boxed{P_{max} = 1.3\times10^7\,W}$$

and pulse duration is $\boxed{t_c = 7.8\,ns}$

Note, could just use $P_{max} \approx \frac{E_{out}}{t_c}$

$$= \frac{E_{flash}}{t_c}$$

$$= \frac{0.1\,J}{7.8\,ns} = 1.3\times10^7\,W$$

4. For mode locked laser,

$$E_{out} = \sum_n E_n e^{i\omega_n t}$$

$$\omega_n = \omega_0 + \delta n$$

$$\delta = 2\pi \Delta \nu_L$$

We are given $E_n \propto \sqrt{P(\nu_n)}$

$$E_n = A e^{-\frac{n^2}{2}\left(\frac{\Delta\nu_L}{\sigma}\right)^2}$$

So, $$E_{out}(t) = A e^{i\omega_0 t} \sum_n e^{-\frac{n^2}{2}\left(\frac{\Delta\nu_L}{\sigma}\right)^2} e^{in\delta t}$$

If $\Delta\nu_L \ll \sigma$, then terms of sum vary slowly, approximate by integral:

$$E_{out}(t) = A e^{i\omega_0 t} \int_{-\infty}^{\infty} dn \, e^{-\frac{n^2}{2}\left(\frac{\Delta\nu_L}{\sigma}\right)^2} e^{in\delta t}$$

Know Fourier transform of Gaussian:

$$\int_{-\infty}^{\infty} dt \, e^{-t^2/\tau^2} e^{i\omega t} = \tau\sqrt{\pi} \, e^{-\omega^2\tau^2/4}$$

Here $t \leftrightarrow n$

$\tau = \sqrt{2}\,\frac{\sigma}{\Delta\nu_L}$

$\omega \to \delta t$

So $$E(t) = A e^{i\omega_0 t} \sqrt{2\pi}\,\frac{\sigma}{\Delta\nu_L}\, e^{-\frac{\delta^2\sigma^2}{2\Delta\nu^2} t^2}$$

and $P_{out}(t) \propto |E|^2$

$$= P_0 \, e^{-\frac{\delta^2 \sigma^2}{\Delta\nu^2} t^2}$$

But $\delta = 2\pi \Delta\nu$

So $$\boxed{P = P_0 \, e^{-(2\pi\sigma)^2 t^2}}$$

Gaussian with $\frac{1}{e}$ duration

$$2 \times \frac{1}{2\pi\sigma} = \frac{1}{\pi\sigma}$$

$$\approx \frac{1}{\Delta\nu_{transition}}$$