Phys 532 Midterm Exam Due March 5

Instructions:

This is a take home, unlimited time exam. You may not discuss the problems with other students, but you are free to use the textbook, your notes, and any other reference materials you wish. You may use a computer for numerical calculations if you want, but you should not expect to need anything more than a calculator.

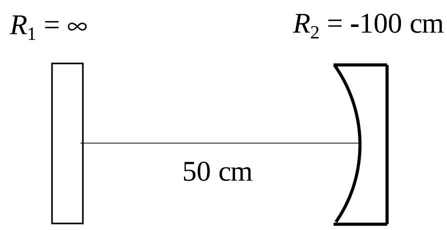
The exam consists of five problems, along the lines of your homework problems. Problem 1 is worth 20 points, and the others are worth 10 points each. (This is because I think problem 1 is important, not because I think it is especially hard.) Partial credit will be given. You will get more credit if I can understand what you are doing, so be sure to explain your methods and copy your solutions out clearly.

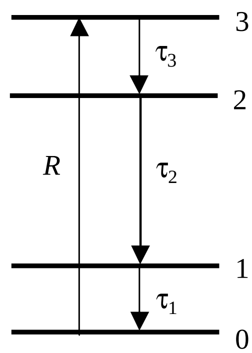
The exam is due at 5:00 PM sharp on Friday March 5.

1. Suppose a laser is constructed using a cavity as shown, with one mirror flat and the other mirror curved with radius $R = -100$ cm. The spacing between mirrors is $d = 50$ cm. The curved mirror will serve as the output coupler, and has a transmission T of 2%. Assume that no other cavity losses are present.

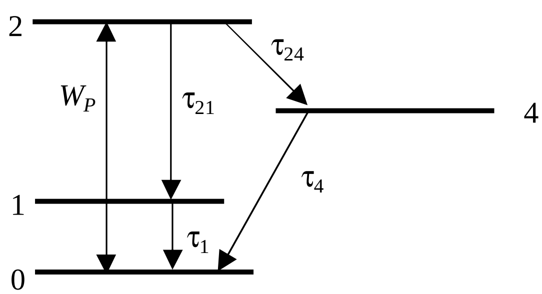
The cavity is filled with a gas having a molecular weight of 200 g/mole. The gas pressure is 1 atmosphere, and it is at room temperature. For these conditions, elastic collisions occur at a rate of 10^9 s^{-1}.

The gas behaves as an ideal four-level laser medium with equal degeneracies. Lasing occurs on the $2 \to 1$ transition which has a wavelength of 5 μm. The decay times are $\tau_3 = 1$ μs, $\tau_2 = 1$ ms, and $\tau_1 = 1$ μs. The $0 \to 3$ transition is weakly pumped at a rate $W_p = 10^{-6}$ s^{-1} per atom. Estimate the total output power produced by the laser.

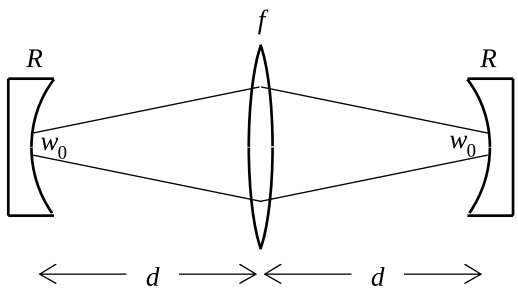




2. Consider a laser medium with energy levels as shown. Here level 4 represents a trapping state: at high pump rates, a large fraction of the atoms can be trapped in level 4, which reduces the available population inversion. You may assume that all levels are equally degenerate. If the $0 \leftrightarrow 2$ transition is pumped at a rate of W_p per atom, calculate the steady-state population inversion $\Delta N = N_2 - N_1$ in terms of the total density N_a.



3. Consider an optical cavity constructed as shown, with two symmetrical curved mirrors of radius R with a thin lens of focal length f placed midway in between. The mirrors are separated by a distance $2d$. It is desired for the Gaussian mode of this cavity to have a focus on the surface of each mirror with specified beam waist w_0, for for light of wavelength λ. Find possible values of R, d, and f such that the cavity satisfies this requirement. Does this configuration result in an optically stable cavity?



4. Suppose a laser medium has a small signal-gain coefficient γ_0 and length ℓ such that $\gamma_0\ell > 1$. Then our simple formulas for the gain threshold and output power are not valid. We showed, however, in a homework problem that the intensity of light changes in the medium according to

$$\ln \frac{I(z)}{I(0)} + \frac{I(z) - I(0)}{I_s} = \gamma_0 z$$

Here $I(z)$ is the intensity after propagating a distance z through the medium, and I_s is the saturation intensity which you may take as given.

Starting from here, determine a formula for the threshold gain coefficient γ_t and the laser output power P_{out}. (Note, γ_t refers to the required small-signal coefficient, even though the the medium might actually be saturated when the laser just starts to oscillate.) Assume that the medium is in a ring cavity and that only modes traveling in one direction can oscillate. The cavity has an output coupler with transmission T which is not small, but assume other cavity losses are negligible. The beam width in the medium is w.

5. Suppose an ideal four-level laser system (as in problem 1) is operating well above threshold, so the small signal gain g_0 is much larger than the total loss $L + T$. If the spontaneous decay time $t_s = \tau_2$ were suddenly increased by a factor of two, how would the laser output power change? Assume nothing else about the system changes. (In particular, assume that the laser transition is collisionally or inhomogeneous broadened, as is usually the case.) Similarly, how does the power scale with the transition wavelength λ?