Phys 532 Assignment 10 Due April 9

1. **Classical Model for Nonlinear Response:** The nonlinear optical response of a medium can be understood in terms of a simple classical model. Suppose a medium contains classical particle-like electrons that move in a 1-dimensional potential

$$V(x) = V_0 \left(\frac{1}{2} \frac{x^2}{a^2} + \frac{1}{3} \frac{x^3}{a^3} \right)$$

where V_0 is a characteristic atomic energy scale and a is a characteristic atomic length scale. Assuming each electron has an electric dipole moment $-ex$, the macroscopic polarization of the medium will be given by $P = -exN$ for electron density N.
(a) Write out the equation of motion for an electron of mass m in this potential which is also driven by an electric field $E(t) = E_0 \exp(i\omega t)$. What is the resonant frequency ω_0 in the limit of small excursion x?
(b) Assuming a steady-state solution of the form

$$x(t) = x_1 e^{i\omega t} + x_2 e^{2i\omega t} + x_3 e^{3i\omega t} + \ldots$$

solve for the amplitudes of the first three terms x_1, x_2, and x_3.
(c) For most optical materials, the lowest resonant frequencies are in the ultraviolet, so we can take $\omega \ll \omega_0$. Evaluate the amplitudes in this limit, and find expressions for the linear susceptability χ and the second and third order nonlinear coefficients, defined by

$$P(t) = \epsilon_0 \chi E + 2dE^2 + 4\chi^{(3)} E^3$$

Note that for $eE_0 \ll V_0/a$, the terms in the expression for P decrease in magnitude as their order increases.

2. **Nearly Degenerate Three-Wave Mixing:** The fundamental equation for second-order nonlinear response is

$$P(t) = 2dE(t)^2$$

for real electric field E and polarization P. (Assume here that the fields can be treated as scalars.) In class, we showed that for a single applied field oscillating at frequency ω, the complex amplitude of the polarization component at 2ω is

$$P(2\omega) = dE(\omega)^2,$$

but for a applied field containing components at distinct frequencies ω_1 and ω_2, the polarization component at $\omega_1 + \omega_2$ is

$$P(\omega_1 + \omega_2) = 2dE(\omega_1)E(\omega_2).$$

Now suppose that the frequencies ω_1 and ω_2 are nearly identical, with $\omega_1 = \omega_2 + \epsilon$. Then on time scales $t \ll 1/\epsilon$, the frequencies $2\omega_1$, $2\omega_2$, and $\omega_1 + \omega_2$ cannot be distinguished. Show that the net polarization at these frequencies satisfies $P = dE^2$ for $E = E(\omega_1) + E(\omega_2)$, just as would be obtained if $\omega_1 = \omega_2$. This should illustrate the continuity between the degenerate and non-degenerate cases.

3. **SHG in Te:** Design a second-harmonic generation experiment in Te using an input at $\lambda = 10.6\ \mu$m. Te is a uniaxial crystal with indices of refraction

λ	n_o	n_e
5.3 μm	4.856	6.307
10.6 μm	4.794	6.243

It has symmetry class 32, which gives three non-zero second-order coefficients, $d_{11} = -d_{12} = -d_{26} = 5.7 \times 10^{-21}$ C/V^2.

Find the phase-matching angle and decide on the proper beam polarization and crystal orientation for maximum power output at 5.3 μm. Find the effective value of the nonlinear coupling parameter, d' (including angle effects).

4. **Properties of BBO:** Look up the properties of the nonlinear crystal β-BaB_2O_4, commonly called BBO. Find the range of wavelengths over which it is transparent, whether it is uniaxial or biaxial, and values for as many of its nonzero second-order coefficients as you can find (including crystal symmetry effects). Also find its various indices of refraction at 1064 nm and 532 nm. Cite the sources you use.

822 students only:

5. **Electro-optic and Nonlinear Optic Coefficients:** In Section 19.2-B, Saleh and Teich explain the relationship between the electro-optic and second-order nonlinear optical coefficients, for scalar (rather than vector) fields. Show that in the vector case, the relationship is

$$r_{ijk} = -\frac{4\epsilon_0 d_{ijk}}{\epsilon_{ii}\epsilon_{jj}}$$

where ϵ_{ij} is the dielectric tensor. Problem 19.6-3 in the text has a hint you may use, but I'd like you to derive the relation given there.