Phys 532 Assignment 12 Due April 30

1. **OPO output power:** Suppose an optical parametric oscillator is constructed using a 1-cm long BBO crystal that is pumped by the third harmonic of a Nd:YAG laser at 355 nm. The crystal angle is set to $\theta = 37°$, which is the phase matching angle for outputs at $\lambda_1 = 590$ nm and $\lambda_2 = 890$ nm. This is a type I phase matching scheme using ordinary output beams and the $d_{21} = 2 \times 10^{-23}$ C/V^2 nonlinear element. The λ_1 light oscillates in a cavity with an internal loss of $L = 5\%$ per round trip and an output coupling transmission $T = 5\%$. Assuming the cavity mode has an optimum focal spot size, find the threshold pump power. If a pulsed pump laser with peak power of 1 kW is used, estimate the power at λ_1 produced. The data you need for this and problem 2 are: $n_{1o} = 1.671$, $n_{2o} = 1.660$, $n_{3o} = 1.720$, $n_{3e} = 1.586$, $dn_1/d\lambda \approx dn_2/d\lambda = 3.3 \times 10^{-5}$ nm^{-1}.

2. **OPO tuning:** The output frequency of an OPO can be tuned by adjusting the crystal angle θ. An expression for the tuning sensitivity $d\lambda_1/d\theta$ can be found through the following argument:

(a) Phase matching requires that

$$n_3\omega_3 = n_1\omega_1 + n_2\omega_2 \tag{1}$$

where the pump frequency ω_3 is fixed and the output frequencies ω_1 and ω_2 can vary. Assume that the output beams are both ordinary, so that only n_3 depends explictly on θ. However, n_1 and n_2 do depend on ω_1 and ω_2, which themselves change with θ. Show then that taking the θ derivative of (1) yeilds

$$\omega_3 \frac{dn_3}{d\theta} = \left[n_1 - n_2 + \omega_1 \frac{\partial n_1}{\partial \omega} - \omega_2 \frac{\partial n_2}{\partial \omega} \right] \frac{d\omega_1}{d\theta}$$

(b) Show now that

$$\frac{dn_3}{d\theta} = \frac{n_3^3}{2} \left(\frac{1}{n_{3o}^2} - \frac{1}{n_{3e}^2} \right) \sin 2\theta$$

Combine these results to get an expression for $d\omega_1/d\theta$.

(c) For the BBO setup of problem 1, what change in θ is required to change λ_1 by 10 nm?

3. **Laser gain as 3rd order process:** In a medium with index of refraction n and gain coefficient γ, the electric field propagates as

$$E(z) = E_0 \exp\left(-ik_0 nz + \frac{\gamma}{2}z\right) \equiv \exp(-i\tilde{n}k_0 z)$$

where $\tilde{n} = n + i\gamma/2k_0$ is called the complex index of refraction. You may assume that $\gamma/k_0 \ll 1$. The complex index is related to the electric susceptabilty as usual, with $\tilde{n}^2 = 1 + \chi$; thus χ is also complex. Recall that for an ideal laser, the gain coefficient is

$$\gamma = \frac{\lambda^2}{8\pi t_s \Delta\nu}\Delta N$$

where the inversion ΔN is given by

$$\Delta N = t_s N_a \frac{\lambda'^2}{8\pi t_s' \Delta\nu'} \frac{I'}{\hbar\omega'}$$

for an optically pumped medium. Here unprimed quantities refer to the laser transition (at frequency ω), while primed quantities refer to the pump transition (frequency ω'). Show that the gain can be interpreted as a third-order nonlinear optical effect, and solve for the effective nonlinear coefficient $\chi^{(3)}$ for the laser medium.

4. **Step-index fiber:** Suppose a step-index fiber has a V parameter of 2.0 for a wavelength $\lambda_0 = 1\ \mu$m. The core radius is $a = 1.5\ \mu$m, and the core index is $n_1 = 1.50$. Since $V < 2.405$, only a single mode is supported. For this mode, find:
(a) the mode parameters k_T and γ, and
(b) the group velocity $v = (d\beta/d\omega)^{-1}$.
You will need to do this problem on a computer.