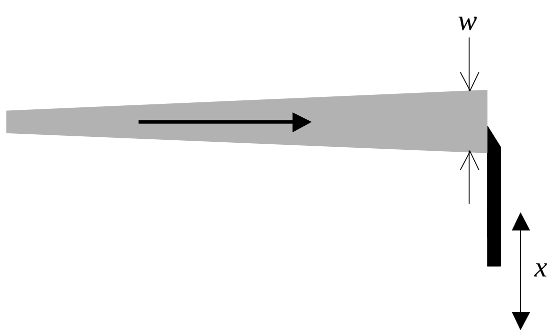
Phys 532 Assignment 2 Due January 30

1. **Gaussian Beam Identification.** (Saleh and Teich Problem 3.1-2) A Gaussian beam of wavelength $\lambda = 10.6\ \mu$m (emitted by a CO_2 laser) has widths $W_1 = 1.699$ mm and $W_2 = 3.38$ mm at two points separated by a distance $d = 10$ cm. Determine the possible locations of the waist and the waist radius.

2. **Measurement of Beam Width.** One way to determine the beam width W of a Gaussian beam is to intercept the beam with a razor blade (or other straight edge) which is mounted on a mechanical translation stage, as sketched below. The center of the beam lies at an unknown position x_0. Suppose the blade is found to block 30% of the incident power at position x_1, and 70% of the power at position x_2. Calculate W in terms of the difference $\Delta = x_2 - x_1$. Hint: the error function $\mathrm{erf}(x)$ is defined by

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

The error function can be evaluated by computer programs like Mathematica and MatLab, or tabulated values can be found in numerous mathematical references.



3. **Gaussian Beams and Ray Matrices.** Suppose that when a Gaussian laser beam passes through an optical system, its complex beam radius q is modified according to

$$q_{\mathrm{out}} = \frac{A_1 q_{\mathrm{in}} + B_1}{C_1 q_{\mathrm{in}} + D_1},$$

while a second system modifies q as

$$q_{\mathrm{out}} = \frac{A_2 q_{\mathrm{in}} + B_2}{C_2 q_{\mathrm{in}} + D_2}.$$

Show that a beam passing consecutively through the two systems will have

$$q_{\mathrm{out}} = \frac{A_3 q_{\mathrm{in}} + B_3}{C_3 q_{\mathrm{in}} + D_3}$$

with

$$\begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}$$

This establishes the correspondence between ray matrices and Gaussian beam propagation.

4. **Imaging a Gaussian Beam.** Suppose a Gaussian laser beam with $\lambda = 532$ nm is collimated with a beam waist of 50 μm at the point $z = 0$. If a a thin lens with focal length 25 mm is placed at $z = 35$ mm, find the position and beam waist for the resulting focus. Compare the focal position with that predicted by geometrical optics for (*i*) a collimated input beam and (*ii*) input light diverging from a focus at $z = 0$.

5. **Retro-reflection of a Gaussian Beam.** A Gaussian laser beam with wavelength 670 nm is focused with a beam waist of 200 μm, and then reflects off a mirror located 10 cm away. What radius of curvature for the mirror is required to have the reflected beam refocus to the same point? Compare your result to the radius of curvature of the beam itself at the location of the mirror.

822 students only:

6. **Hermite-Gaussian Beams.** Read Saleh and Teich Section 3.3. You should notice a similarity between the Hermite-Gaussian beam solutions and the eigenfunctions of a simple harmonic oscillator potential from quantum mechanics. This relationship can be established by considering solutions to the paraxial wave equation

$$\nabla_T^2 A - 2ik\frac{\partial A}{\partial z} = 0$$

that have the form

$$A(x, y, z) = f(x, y)N(z)e^{-\alpha(z)\rho^2}.$$

(a) Show that at any z, the equation for A is formally equivalent to that of the 2D quantum harmonic oscillator in x and y.

(b) Determine how the spatial width of the beam in the x direction scales with the order l. Hint: Use the analogy established in (a) and what you already know about harmonic oscillators.