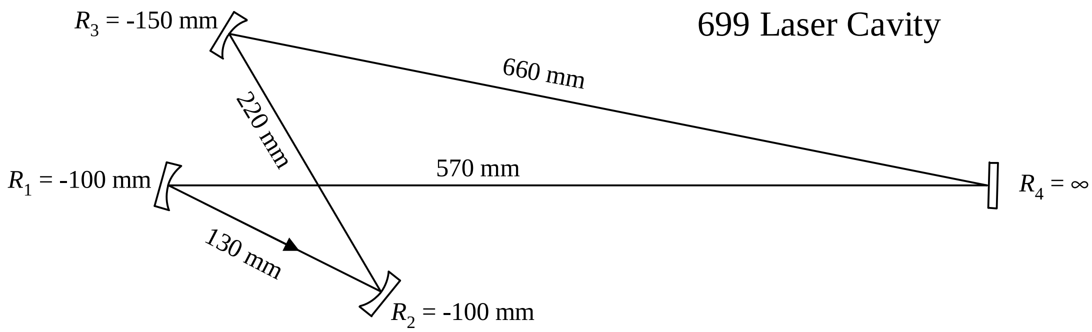
Phys 532 Assignment 3 Due February 6

1. **Modes of 699 Cavity:** (a) Find the Gaussian beam mode of the the 699 laser cavity. In particular, locate all the positions in the cavity at which a focus occurs, and determine the beam waist at each focus. Assume a wavelength of 800 nm.
(b) In this laser, the mirror labeled R_4 is only partially reflecting, to allow a fraction of the circulating light to escape. What is the divergence angle of the output beam?



2. **Mode Frequencies of 699 Cavity:** Find the free spectral range ν_F and the round-trip Guoy phase shift $\Delta\zeta$ for the 699 cavity. (Ignore any phase shifts due to the mirrors.)

3. **Linewidth of 699 Cavity:** If mirrors R_1, R_2, and R_3 of the 699 cavity have (intensity) reflectances of 99.5%, and mirror R_4 has reflectance 95%, calculate the linewidth $\delta\nu$, the finesse $\mathcal{F}$, the quality factor Q, the photon lifetime τ_p, the loss per pass Γ, and the distributed loss coefficient α for the cavity.

4. **Linewidth of a Harmonic Oscillator:** I stated in class that there is a general relationship between the quality factor Q and the linewidth of an oscillator. Establish this relation for the case of a damped harmonic oscillator, governed by the differential equation

$$m\ddot{x} + b\dot{x} + kx = F(t)$$

for mass m, damping coefficient b, spring constant k, and driving force $F(t)$.
(a) First, consider the undriven oscillator, where $F = 0$. Show that the general solution can be written as

$$x(t) = Ae^{-\gamma t/2}\cos(\bar{\omega}t + \phi)$$

for arbitrary A and ϕ. Find expressions for γ and $\bar{\omega}$.

(b) Using this solution, calculate the energy in the oscillator,

$$E(t) = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2.$$

(c) Assuming a weakly damped oscillator in which $\gamma \ll \bar{\omega}$, calculate

$$Q' = -\frac{\bar{\omega}\langle E\rangle}{d\langle E\rangle/dt}$$

where $\langle \ldots \rangle$ indicates an average over one period of the motion.
(d) Now consider a driven oscillator, with $F(t) = F_0 e^{i\omega t}$. Find the steady-state solution for $x(t)$ in this case. ("Steady-state" means the form the solution takes after any possible transients described by the solution of part (a) have decayed to zero.)
(e) Again assuming that $\gamma \ll \bar{\omega}$, calculate the linewidth $\Delta\omega$ of the oscillator, defined as the full-width at half-max of the response $|x|^2$ as a function of drive frequency ω.
(f) Show that the quality factor Q, defined by $\omega/\Delta\omega$, is equal to the quantity Q' calculated in part (c).

5. **Photons in a Laser Cavity:** A symmetrical linear cavity has mirrors with radius of curvature $R = -10$ cm separated by $d = 10$ cm. The cavity contains $N = 10^4$ photons at a wavelength $\lambda = 500$ nm. Calculate the peak intensity in the cavity. Note: in general, the energy in a volume V is given by

$$\mathcal{E} = \frac{1}{c}\int_V I(\mathbf{r})dV$$

where $I(\mathbf{r})$ is the intensity at point $\mathbf{r}$. Be careful, because here I is the total intensity, which in this case comes from the sum of two waves (the forward and backward propagating beams).

822 students only:

6. **Transverse Mode Spacing of the 699 Cavity:** Read Saleh and Teich section 9.2D on pages 336-337. Note, however, that their variable $\Delta\zeta$ is not quite the same as what we defined in class.
(a) In the 699 cavity, what is the frequency spacing between transverse modes?
(b) Suppose an external laser beam is injected into the cavity through one of the mirrors in such a way that the beam is coupled strongly to the $(0, 0)$ mode, but the coupling to the $(l + 1, m)$ and $(l, m + 1)$ modes is only half as strong as the coupling to the (l, m) mode. (This type of behavior occurs when the injected beam is similar to, but not the same as, the Gaussian mode of the cavity.) Sketch a plot showing the power coupled into the cavity as a function of frequency in this case.