Phys 532 Assignment 4 Due February 13

1. **Stimulated vs. Spontaneous Emission:** (a) An atom contains two energy levels connected by a transition with wavelength $\lambda_0 = 0.7\ \mu$m, spontaneous lifetime $t_{sp} = 3$ ms, and a Lorentzian lineshape with width $\Delta\nu = 50$ GHz. The atom is prepared in the upper state, and is placed in a resonator with volume $V = 100$ cm^3 that has a cavity mode at the center frequency ν_0. How many photons must be present in this cavity mode so that the rates for stimulated and spontaneous emission are equal?
(b) If an atom in free space has a transition with frequency ν_0, spontaneous lifetime t_s, and light scattering cross section $\sigma(\nu)$, find a simple expression for the intensity required to make the stimulated transition rate W equal the spontaneous emission rate.

2. **Emission into a Laser Cavity:** Equations (12.2-1), (12.2-3), and (12.2-5) can only be applied to closed cavities with well-defined volumes V. In a laser cavity, confined modes can be characterized by their intensity distribution $I(\mathbf{r})$, and the photon interaction rates can be more generally expressed as, for example,

$$p_{sp} = \frac{I(\mathbf{r})c\sigma(\nu)}{\int I(\mathbf{r})dV}$$

for spontaneous emission. Consider an atom positioned at the center of a symmetric cavity with mirror spacing d and radii of curvature such that the TEM$_{00}$ (Gaussian mode) beam waist is W_0. The atom is in the upper state of a transition with oscillator strength S. Calculate the probability that a particular photon emitted by the atom enters a TEM$_{00}$ mode. You can assume that the free spectral range of the cavity is small compared to the atomic transition linewidth and that the total spontaneous emission rate P_{sp} is not altered by the cavity.

3. **Inhomogeneous Broadening:** Suppose a cell of length L contains a gas of atoms having a transition at frequency ν_0 and linewidth $\Delta\nu$. The cell is placed in a spatially varying magnetic field B with $B(z) = \beta z$ for $z = 0$ at the center of the cell. Through the Zeeman effect, the magnetic field shifts the frequency of the transition according to $\nu_0' = \nu_0 + \mu B$, for some constant μ. If $\Delta\nu$ is small compared to $\mu\beta L$, what is the lineshape function $\bar{g}(\nu)$ for the gas?

4. **Absorption Coefficient:** Sodium atoms have a doublet of excited states labeled $3p_{1/2}$ and $3p_{3/2}$ that are connected to the $3s_{1/2}$ ground state by transitions at 589.6 nm and 589.0 nm respectively. The excited states have a radiative lifetime of 16 ns and degeneracies of 2 and 4, while the ground state has degeneracy 2. At a temperature of 400 K, Na vapor has a number density $N = 3 \times 10^{11}$ cm^{-3} and the collision rate between atoms is of order 1000 s^{-1}. Identify the dominant source of line broadening in this case, and then find the peak absorption coefficient for each of the two lines.

822 students only:

5. **Quantum Calculation of σ:** Transitions rates in quantum mechanics can be calculated using perturbation theory, which yields Fermi's Golden Rule:

$$R_{2\to 1} = \frac{4\pi^2}{h} |\langle 1|H|2\rangle|^2 \delta(E_2 - E_1 - h\nu)$$

where R_{12} is the rate to make a transition from state $|2\rangle$ to state $|1\rangle$ while emitting a photon of frequency ν into a specific field mode. Here E_2 and E_1 are the respective state energies and and H is the Hamiltonian. If you haven't seen this yet, you will later this semester in your quantum class.

(a) Use Fermi's Golden rule to express the cross section σ in terms of the matrix element $\langle 1|H|2\rangle$.

(b) For an electric dipole transition, the matrix element can be expressed as

$$\langle 1|H|2\rangle = e\sqrt{\frac{h\nu}{2V\epsilon_0}}\hat{\epsilon}\cdot \mathbf{d}_{12}$$

where ϵ_0 is the permitivity of free space, $\hat{\epsilon}$ is the polarization vector of the light, and and $\mathbf{d}_{12}$ is the position matrix element $\langle 2|\mathbf{r}|1\rangle$ for the electronic charge. Express σ in terms of $\mathbf{d}_{12}$. You can simplify the answer using the fine structure constant $\alpha = e^2/2\epsilon_0 hc$.

Note that the δ-function is an artifact of perturbation theory; in a non-perturbative calculation it is replaced by the lineshape function $g(\nu)$.