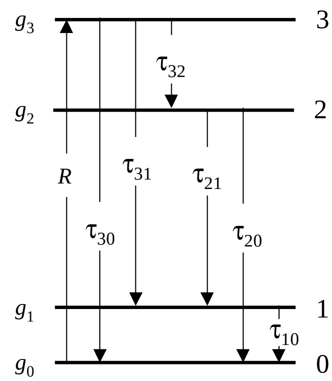
Phys 532 Assignment 5 Due February 20

1. **Rate Equations:** Use the rate equations to calculate the population inversion $\Delta N_0 = N_2 - (g_2/g_1)N_1$ for the four-level system shown. The only transition being driven is the $0 \leftrightarrow 3$ pump transition, and you can assume that the pump rate is low enough to neglect depletion of state 0. Pumping can therefore be characterized by the total rate R (units of (atoms/s)/m^3), as shown. Feel free to simplify expressions by using the total lifetimes τ_i for each state. For instance,

$$\frac{1}{\tau_2} = \frac{1}{\tau_{21}} + \frac{1}{\tau_{20}}.$$



2. **Gain Coefficient:** Suppose that the system of problem 1 describes a solid state laser medium with the following data:

$\lambda_{03} = 500$ nm	$\lambda_{12} = 800$ nm	$\Delta\nu_{03} = 5 \times 10^{13}$ Hz
$\Delta\nu_{12} = 10^{11}$ Hz	$\tau_{30} = \tau_{32} = 100$ ns	$\tau_{31} = 2$ ms
$\tau_{21} = 1$ ms	$\tau_{20} = 1$ ms	$\tau_{10} = 25$ ns
$g_0 = g_1 = 1$	$g_2 = 2$	$g_3 = 4$

Take the $0 \leftrightarrow 3$ and $1 \leftrightarrow 2$ transitions to be radiative, so that τ_{ij} gives the appropriate spontaneous emission lifetime. The medium consists of ions in a 1-cm long crystal, where the total density of ions is $N_a = 10^{19}$ cm^{-3} and the index of refraction of the host crystal is $n = 1.7$. What pumping intensity on the $0 \leftrightarrow 3$ transition is required to achieve a total gain through the crystal of 2.5? Neglect any saturation effects.

3. **Saturation Intensity:** In the system of problem 1, take $\tau_{20} = \tau_{31} = \infty$ and calculate the saturation intensity for the $2 \rightarrow 1$ transition. Recall that saturation intensity I_s is defined by

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1 + I/I_s}$$

for gain coefficient γ in the presence of radiation with intensity I.

4. **Saturation Intensity for Sodium:** As in problem 4 from assignment 4, sodium atoms have a doublet of excited states labeled $3p_{1/2}$ and $3p_{3/2}$ that are connected to the $3s_{1/2}$ ground state by transitions at 589.6 nm and 589.0 nm respectively. The excited states have a radiative lifetime of 16 ns and degeneracies of 2 ($3p_{1/2}$) and 4 ($3p_{3/2}$), while the ground state has degeneracy 2. At a temperature of 400 K, the vapor density is low enough that collisions are negligible. Calculate the saturation intensity at $\nu = \nu_0$ for each of the two transitions.

822 students only:

5. **Saturated Gain in a Inhomogeneous Medium:** Read Saleh and Teich section 13.3C. As in problem 3 from assignment 4, consider a cell of length L containing a gas of atoms with density N_a, and with a transition at frequency ν_0 with spontaneous lifetime t_s and homogeneous linewidth $\Delta\nu$. The cell is placed in a spatially varying magnetic field B with $B(z) = \beta z$ for $z = 0$ at the center of the cell. The magnetic field shifts the frequency of the transition according to $\nu_0' = \nu_0 + \mu B$, for some constant μ. Take the lower level to be the ground state, and assume no pumping mechanism is present. The levels have equal degeneracies. Determine the inhomogeneous absorption coefficient $\bar{\alpha}(\nu) = -\bar{\gamma}(\nu)$ for light of intensity I, in the limit $\Delta\nu \ll \mu\beta L$. Assume sufficent light is present to saturate the transition, but not greatly more than that.