Phys 532 Assignment 11 Solutions

1. In general, have $P(2\omega) = k\, P(\omega)^2$

for constant k

If CW conversion efficiency is ε,

know $\varepsilon = k\, P_{cw}$

For pulsed laser, peak power $P_{pulse} = P_{cw} \cdot \dfrac{T}{\Delta t}$ (same average power)

So
$$P_p(2\omega) = k\, P_p(\omega)^2$$
$$= k\left(\frac{T}{\Delta t}\right)^2 P_{cw}^2$$
$$= \varepsilon \left(\frac{T}{\Delta t}\right)^2 P_{cw}$$
$$= \varepsilon \left(\frac{T}{\Delta t}\right) P_p(\omega)$$

So, new conversion efficiency is $\boxed{\varepsilon \times \frac{T}{\Delta t} \gg \varepsilon}$

2. a) With $d_{31} = d_{zxx}$, need ordinary beams at ω, extraordinary at 2ω

So, need $n_e(550\,\text{nm}) = n_o(1100\,\text{nm})$

Plot relations vs. T:

Find solution at $T \approx 397\,\text{K}$
$= 124\,°\text{C}$

b) In class derived

$$P(\omega_3) = \frac{8}{\pi} \mathfrak{Z}_0^3 \frac{n_{min}\,\omega_3^2}{n_1 n_2 n_3} \omega_{min} \frac{L}{c_0} d'^2 P(\omega_1) P(\omega_2)$$

Here $d' = \frac{1}{2} d_{31}$, due to $\omega_1 = \omega_2$

$n_1 = n_2 = n_3 = n_{min} = 2.23$

$\omega = \frac{2\pi c_0}{\lambda}$ $\quad 2\omega = 2 \cdot \frac{2\pi c_0}{\lambda}$ $\quad \lambda = 1.1\,\mu m$

$$P(2\omega) = \frac{8}{\pi} \mathfrak{Z}_0^3 \frac{1}{n^2} 4 \left(\frac{2\pi c_0}{\lambda}\right)^3 \frac{L}{c_0} \frac{1}{4} d_{31}^2 P(\omega)^2$$

$$= 64\pi^2 \mathfrak{Z}_0^3 \frac{1}{n^2} \frac{c_0^2 L}{\lambda^3} d_{31}^2 P(\omega)^2$$

$$= 64\pi^2 (377\Omega)^3 \frac{1}{2.23^2} \cdot \frac{(3\times10^8 \frac{m}{s})^2}{(1.1\,\mu m)^3} \cdot 0.03\,m \cdot (4\times10^{-23} \tfrac{C}{V^2})^2 (0.1\,W)^2$$

$$= \boxed{220\,\mu W}$$

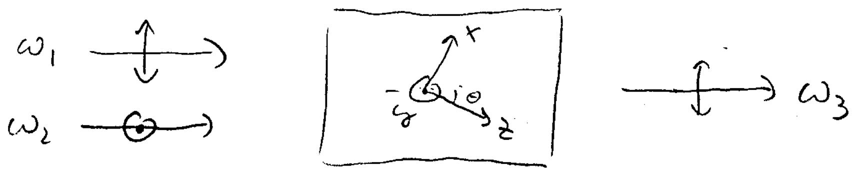
3. a) $\frac{1}{\lambda_3} = \frac{1}{\lambda_1} - \frac{1}{\lambda_2}$

max λ_3: $\frac{1}{\lambda_3} = \frac{1}{683\,nm} - \frac{1}{805\,nm} = \frac{1}{4.5\,\mu m}$

min λ_3 = $\frac{1}{677\,nm} - \frac{1}{811\,nm} = \frac{1}{4.1\,\mu m}$

So λ_3 tunable from 4.1 to 4.5 μm

b) Using $d_{16} = d_{xxy}$ coefficient. So have



(ie, $\phi = 0°$)

c) Need $\dfrac{n_e'(\omega_1)}{\lambda_1} - \dfrac{n_o(\omega_2)}{\lambda_2} = \dfrac{n_e'(\omega_3)}{\lambda_3}$ $(k_3 = k_1 - k_2)$

or

$$\frac{1}{\lambda_1}\left[\frac{\cos^2\theta}{n_o(\omega_1)^2} + \frac{\sin^2\theta}{n_e(\omega_1)^2}\right]^{-1/2} - \frac{n_o(\omega_2)}{\lambda_2}$$

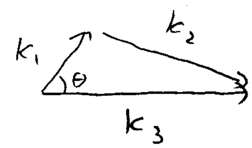
$$= \frac{1}{\lambda_3}\left[\frac{\cos^2\theta}{n_o(\omega_1)^2} + \frac{\sin^2\theta}{n_e(\omega_1)^2}\right]^{-1/2}$$

Solve by plotting numerically, Find $\theta = 40°$

Since we use $E_x(\omega_1)$ and $E_x(\omega_3)$, get

$$d' = d_{16}\cos^2\theta$$
$$= 0.59\, d_{16}$$

4. Phase matching triangle:



$$\frac{1}{\lambda_2} = \frac{1}{\lambda_3} - \frac{1}{\lambda_1} = \frac{1}{0.725\,\mu m}$$

Using d_{31} coefficient, we need output fields polarized along x

So $|\vec{k}_1| = n_o(\omega_1)\, k_{10}$

$|\vec{k}_2| = n_o(\omega_2)\, k_{20}$

Have $n_o(2\mu m) = 2.20 = n_1$

$n_o(0.725\mu m) = 2.27 = n_2$

$n_e(0.532\mu m) = 2.23 = n_3$

Law of cosines gives

$$2k_1k_3\cos\Theta = k_1^2 + k_3^2 - k_2^2$$

$$\cos\Theta = \frac{\left(\frac{n_1}{\lambda_1}\right)^2 + \left(\frac{n_3}{\lambda_3}\right)^2 - \left(\frac{n_2}{\lambda_2}\right)^2}{2\left(\frac{n_1}{\lambda_1}\right)\left(\frac{n_3}{\lambda_3}\right)}$$

$$= 0.9735$$

$$\boxed{\Theta_1 = 13^\circ}$$

$\Theta_2 = 4.6^\circ$

5. a) Need to find normal to ellipse

Use gradient:

$$\hat{S} \propto \vec{\nabla}_k \left(\frac{k_x^2}{n_o^2} + \frac{k_z^2}{n_e^2} \right)$$

$$= \frac{2k_x}{n_o^2} \hat{x} + \frac{2k_z}{n_e^2} \hat{z}$$

$$= 2k_o n(\theta) \left[\frac{\sin\theta}{n_o^2} \hat{x} + \frac{\cos\theta}{n_e^2} \hat{z} \right]$$

Then $\tan\alpha = \frac{S_x}{S_z} = \frac{\sin\theta}{n_o^2} \cdot \frac{n_e^2}{\cos\theta} = \frac{n_e^2}{n_o^2} \tan\theta$

$$\boxed{\alpha = \tan^{-1}\left(\frac{n_e^2}{n_o^2} \tan\theta \right)}$$

b) $\alpha(\omega) = \tan^{-1}\left[\left(\frac{1.5357}{1.5055} \right)^2 \tan 53.5^\circ \right] = 54.58^\circ$

$\alpha(2\omega) = \tan^{-1}\left[\left(\frac{1.4897}{1.5357} \right)^2 \tan 53.5^\circ \right] = 51.82^\circ$

$$\boxed{\Delta\alpha = 2.76^\circ}$$