Phys 532 Assignment 6 Solutions

1. a) $P = \pi w^2 I_s \, T\left(\frac{g_0}{L+T} - 1\right)$

$$\frac{dP}{dT} = \pi w_s I_s \left[\left(\frac{g_0}{L+T} - 1\right) - T\frac{g_0}{(L+T)^2}\right] = 0$$

$$g_0(L+T) - (L+T)^2 - g_0 T = 0$$

$$(L+T)^2 = g_0 L$$

$$L+T = \sqrt{g_0 L}$$

$$\boxed{T = \sqrt{g_0 L} - L}$$

b) $P = \pi w^2 I_s \left(\sqrt{g_0 L} - L\right)\left[\frac{g_0}{L + \sqrt{g_0 L} - L} - 1\right]$

$$= \pi w^2 I_s \left(\sqrt{g_0 L} - L\right)\left[\sqrt{\frac{g_0}{L}} - 1\right]$$

$$= \pi w^2 I_s \left[g_0 - \sqrt{g_0 L} - \sqrt{g_0 L} + L\right]$$

$$\boxed{P = \pi w^2 I_s \left(\sqrt{g_0} - \sqrt{L}\right)^2}$$

2. a) Know absorption coefficient : $\alpha = \sigma N_a$

So $\sigma_{12} = \frac{g_1}{g_2}\frac{\alpha}{N_a} = \frac{0.1\,\text{cm}^{-1}}{10^{17}\,\text{cm}^{-3}} = 10^{-18}\,\text{cm}^2$

And $\sigma_{12} = \frac{\lambda^2}{8\pi t_s} g(\nu)$

For collision broadening, $g(\nu)$ is Lorentzian with

$$\Delta\nu = \frac{1}{2\pi}\left(\frac{1}{\tau_2} + 2f_{col}\right)$$

Expect $f_{col} \gg \frac{1}{\tau_2}$, so $\Delta\nu = \frac{f_{col}}{\pi}$

and $g(\nu_0) = \frac{2}{\pi\Delta\nu} = \frac{2}{f_{col}}$

So $t_s = \frac{\lambda^2}{8\pi\sigma_{12}}\frac{2}{f_{col}} = \frac{(570nm/1.3)^2}{8\pi\times10^{-18}cm^2}\frac{2}{10^{12}s^{-1}}$

$$\boxed{t_s = 150\mu s}$$

Indeed, $f_{col} = 10^{12}s^{-1} \gg \frac{1}{t_s} = 6.5\times10^3 s^{-1}$

b) For three-level system, rate equations give

$$\Delta N_0 = \frac{W_p\tau_2 - 1}{W_p\tau_2 + 1} N_a$$

$$\gamma = \sigma\Delta N$$

and $2\gamma_t l = L + T = 0.1$ * Note, typo error on assignment sheet

$$\gamma_t = \frac{0.1}{20cm} = 5\times10^{-3}cm^{-1}$$

need $\Delta N_t = \frac{\gamma_t}{\sigma} = \frac{5\times10^{-3}cm^{-1}}{10^{-18}cm^2} = 5\times10^{15}cm^{-3}$

So $\frac{\Delta N_t}{N_a} = 0.05$,

solve, se $W_p\tau_2 = \frac{1 + \Delta N/N_a}{1 - \Delta N/N_a} = 1.1$

$$W_p = \frac{1.1}{\tau_2} = \frac{1.1}{150\mu s} = 7.4\times10^3 s^{-1}$$

Relate to absorption:

$$W_p = \sigma_{13} \, I_p / h\nu_{13}$$

$$\sigma_{13} = \frac{\alpha_{13}}{N_a} = \frac{10\ cm^{-1}}{10^{17}\ cm^{-3}} = 10^{-16}\ cm^2$$

So $$I_p = W_p \frac{h\nu_{13}}{\sigma_{13}} = 7.4\times10^3\ s^{-1} \frac{(6.63\times10^{-34}\ Js)\left(\frac{3\times10^8\ m/s}{520\ nm}\right)}{10^{-16}\ cm^2}$$

$$\boxed{I_p = 28.3\ \frac{W}{cm^2}}$$

c) $$I_{sat} = \frac{h\nu}{\sigma_{12}\tau_s}$$

For three level system have

$$\tau_s = \frac{2t_{sp}}{1+t_{sp}W_p} \quad (S\&T\ .13.2-25)$$

$t_{sp}W_p = 1.1$ at threshold

$$= \frac{300\mu s}{1+1.1} = 142\mu s$$

$$I_s = \frac{(6.63\times10^{-34}\ Js)\left(\frac{3\times10^8\ m/s}{570\ nm}\right)}{(10^{-18}\ cm^2)(142\mu s)} = 2.457\ \frac{W}{cm^2}$$

d) If $W_p = 2\times W_t$ then $$\Delta N_0 = \frac{W_p\tau_2 - 1}{W_p\tau_2+1} N_a$$

$W_p\tau_2 = 2.2$

$$= \frac{2.2-1}{2.2+1} N_a = 0.375\ N_a$$

$$= 3.75\times10^{16}\ cm^{-3}$$

Then $\gamma = \sigma_{12} \Delta N_0 = 10^{-18}\,cm^2 \cdot 3.75\times10^{16}\,cm^{-3}$

$= 0.0375\ cm^{-1}$

$S_0 = 2\gamma_0 l = 0.75$

Also;

$I_s = \frac{h\nu}{\sigma_{12}}\ \frac{1+t_{sp}W_p}{2t_{sp}}$

$\rightarrow 2457\,\frac{W}{cm^2} \times \frac{3.2}{2.1}$

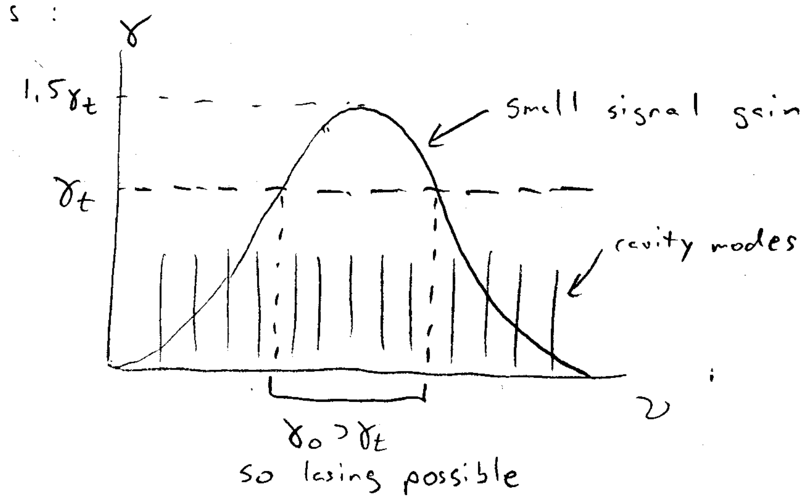
$= 3744\ \frac{W}{m^2}$

Then $P_{out} \simeq \pi w^2 I_s \left(\frac{S_0}{T+L} - 1\right) T$

$= \pi(100\mu m)^2 \times (3744\ \frac{W}{cm^2})\left(\frac{0.75}{0.1} - 1\right)(0.05)$

$\boxed{P_{out} = 0.38\ W}$

3. Picture is:



Doppler profile: $\bar{g}(\nu) = \frac{1}{\sqrt{2\pi}\,\sigma_D} e^{-\frac{(\nu-\nu_0)^2}{2\sigma_D^2}}$

$\sigma_D = \frac{1}{\lambda}\left(\frac{kT}{M}\right)^{1/2}$

$= \frac{1}{514nm}\left(\frac{1.38\times10^{-23}\frac{J}{K}\cdot 2000K}{40\times1.67\times10^{-27}kg}\right)^{1/2}$

$= 1.25\ GHz$

Find range $\gamma > \gamma_t$

Find ν such that $\bar{g}(\nu) = \frac{2}{3}\, g(\nu_0)$

$$e^{-\frac{(\nu-\nu_0)^2}{2\sigma_0^2}} = \frac{2}{3}$$

$$(\nu-\nu_0)^2 = 2\sigma_0^2 \ln 1.5$$

$$\nu-\nu_0 = \pm \sigma_0 \sqrt{2\ln 1.5}$$

$$\nu = \nu_0 \pm \sigma_0 \sqrt{2\ln 1.5}$$

Freq range $\Delta\nu = 2\sigma_0 \sqrt{2\ln 1.5}$

$$= 1.8\,\sigma_0$$

$$= 2.25\ \text{GHz}$$

On other hand, free spectral range of cavity is $\nu_F = \frac{c}{2d} = 3\times 10^8\ \text{Hz} = 0.3\ \text{GHz}$

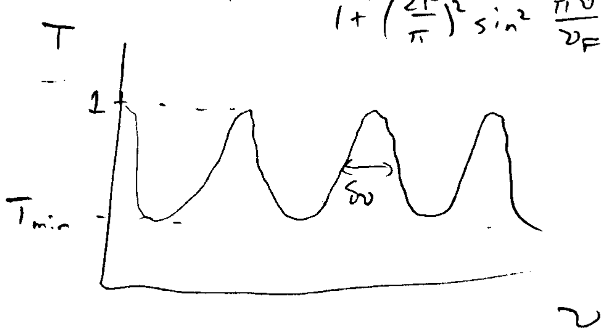
So expect about $\frac{2.25\ \text{GHz}}{0.3\ \text{GHz}} = 7.5$

So, $\boxed{7 \text{ or } 8}$ modes should lase

4. Want to use etalon to suppress all but one mode.

Etalon transmission:

$$T = \frac{1}{1+\left(\frac{2F}{\pi}\right)^2 \sin^2 \frac{\pi\nu}{\nu_F}}$$



$$\nu_F = \frac{c}{2nd}$$

$$T_{min} = \frac{1}{1+\left(\frac{2F}{\pi}\right)^2}$$

So we need:

a) Only one transmission peak in gain range

b) T_{min} low enough to suppress lasing

c) $\delta\nu \approx$ mode spacing of laser cavity
So one & only one mode can oscillate within etalon peak

From a, want $\nu_F = \frac{c}{2nd} \approx 2.25$ GHz

$$d = \frac{3\times10^8 \text{ m/s}}{2\times1.5\times2.25\text{GHz}} = 0.044\text{ m} = 4.4\text{ cm}$$

b): Threshold gain is $\frac{2}{3}$ of peak gain
So, want $T_{min} \le \frac{2}{3}$

Then
$$\frac{1}{1+\left(\frac{2F}{\pi}\right)^2} < \frac{2}{3}$$

$$1+\left(\frac{2F}{\pi}\right)^2 > 1.5$$

$$\left(\frac{2F}{\pi}\right)^2 > \frac{1}{2}$$

$$F > \frac{\pi}{2\sqrt{2}} = 1.1$$

c): In general, FWHM of etalon peak $\delta\nu$

solution of $\left(\frac{2F}{\pi}\right)^2 \sin^2 \frac{\pi\delta\nu}{2\nu_F} = 1$

Want $\delta\nu \sim 0.3$ GHz
$\nu_F = 2.25$ GHz

So $$\frac{\pi\,\delta\nu}{2\nu_F} \approx 0.21$$

So $$\sin^2 \frac{\pi\,\delta\nu}{2\nu_F} \approx \left(\frac{\pi\,\delta\nu}{2\nu_F}\right)^2$$

and $$\left(\frac{2F}{\pi}\right)^2 \Big/ \left(\frac{\pi\,\delta\nu}{2\nu_F}\right)^2 = 1$$

$$F \approx \frac{\nu_F}{\delta\nu} = \frac{2.25\,\text{GHz}}{0.3\,\text{GHz}} = 7.5$$

To satisfy all conditions, take

$$\boxed{\begin{array}{l} d = 4.4\,\text{cm} \\ F = 7.5 \end{array}}$$

5. Have $$\gamma = \frac{\gamma_0}{1 + I/I_s} \approx \frac{\gamma_0 I_s}{I}$$

Then $$\frac{dI}{dz} = \gamma I = \gamma_0 I_s$$

So $$\boxed{I(z) = I_{in} + \gamma_0 z I_s}$$

I grows linearly, so $\boxed{\text{yes,}}$ amplification does occur.

In general, can write

$$\frac{dI}{dz} = \frac{\gamma_0}{1 + I/I_s} I$$

$$\int_{I_{in}}^{I(z)} \frac{1}{I} + \frac{1}{I_s} \, dI = \int_0^z \gamma_0 \, dz$$

$$\ln \frac{I(z)}{I_{in}} + \frac{I(z) - I_{in}}{I_s} = \gamma_0 z$$

But we can't get an explicit formula for $I(z)$.