02/11/05 Lecture 11.

Photon-atom interaction in a cavity

$P_{abs} = \frac{\text{Prob.}}{\text{time}}$: for ground state atom to absorb a photon from an occupied mode

$= N \frac{c}{V} \sigma(\nu)$

$P_{stim} = \frac{\text{Prob.}}{\text{time}}$ for excited atom to emit a photon into occupied mode

$= N \frac{c}{V} \sigma(\nu)$

$P_{spont} = \frac{\text{Prob.}}{\text{time}}$ for excited atom to emit a photon into an empty mode

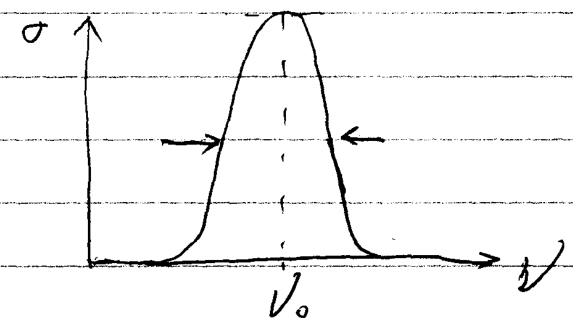
$= \frac{c}{V} \sigma(\nu)$

V: volume σ = light scattering cross section

N: occupation # of mode

Note: P_{spont} + P_{stim} add: $P_{emit} = (N+1) \frac{c}{V} \sigma$

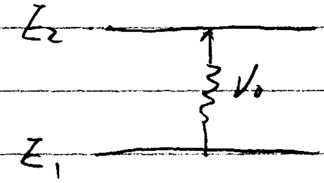
$\sigma(\nu)$ is frequency dependent



Peak at $\nu_0 = \frac{E_2 - E_1}{h}$

Peak $\sigma(\nu_0) = \sigma_0$

width $\Delta\nu$



usuall convenient to define.

oscillator strength $S = \int_0^\infty \sigma(\nu)\, d\nu$

measure overall strength of interaction

and line shap function $g(\nu) = \frac{\sigma(\nu)}{S}$

So $\int g(\nu)\, d\nu = 1$

the $g(\nu_0) \approx \frac{1}{\Delta\nu}$, $\sigma_0 = \sigma(\nu_0) = \frac{S}{\Delta\nu}$

So far how atom interacts with one mode of a cavity, always multiple mode present.

Especially, for sponteneous emission

Total decay rate.

$$P_{sp} = \sum_{\text{mode } i} P_{sp}(i) \longrightarrow \int_0^\infty \left[\frac{c}{V}\sigma(\nu)\right]\left[V M(\nu)\, d\nu\right]$$

$M(\nu)$ = density of modes
= # of modes / frequency range / volume

In cubical cavity, modes labeled by (n_x, n_y, n_z)

frequency $\nu = \frac{c}{d}\sqrt{n_x^2 + n_y^2 + n_z^2}$

# of modes in some range of n's
= "volume" $\Delta n_x \Delta n_y \Delta n_z$

"spherical coordinates" $= 4\pi n^2 \Delta n$ $\quad n = \sqrt{n_x^2 + n_y^2 + n_z^2}$

$$= 4\pi \frac{\nu^2 d^3}{c^3} d\nu$$

$\times 2$ for two polarizations

$$\text{\# of modes} = 8\pi \frac{\nu^2}{c^3} V \, d\nu$$

$$M(\nu) = \frac{\text{\# of modes}}{V \, d\nu} = 8\pi \frac{\nu^2}{c^3}$$

So $P_{sp} = 8\pi \frac{1}{c^2} \int_0^\infty \nu^2 \sigma(\nu)\, d\nu$

Assuming $\Delta\nu \ll \nu_0$, σ is sharply peaked.
$\nu \approx \nu_0$ over $\Delta\nu$

$$P_{sp} = \frac{8\pi}{c^2} \nu_0^2 \int_0^\infty \sigma(\nu)\, d\nu$$

$$= \frac{8\pi}{\lambda_0^2} S$$

Define spontaneous life time t_{sp} by $P_{sp} = \frac{1}{t_{sp}}$

So $$\boxed{S = \frac{\lambda^2}{8\pi t_{sp}}}$$

Also gives $\sigma(\nu) = \frac{\lambda^2}{8\pi t_{sp}} g(\nu)$

Generally use t_{sp} as measure of transition strength
(can be measured experimentally or cal from perturbation theory)

Stimulated process

Stimulation by monochromatic light has basic rate

$$P_{st} = P_{abs} = N \frac{c}{V} \sigma(\nu) \equiv W$$

Usually characterize light by intensity I

$$I : \frac{\text{energy}}{\text{time} \cdot \text{area}}$$

In a cubical cavity   energy $N = h\nu$
area $= d^2$
time $= \frac{d}{c}$

$$I = N h\nu \frac{c}{d^3} = N h\nu \frac{c}{V}$$

So $$W = \left(\frac{I}{h\nu}\frac{V}{c}\right)\left(\frac{c}{V}\sigma\right) = I\frac{\sigma}{h\nu}$$

or $$\boxed{W = I\frac{\lambda^3}{8\pi hc\, t_{sp}} g(\nu)}$$

transitions with broad band light

characterize by energy density $\rho(\nu) : \frac{\text{energy}}{\text{freq. range} \times \text{volume}}$

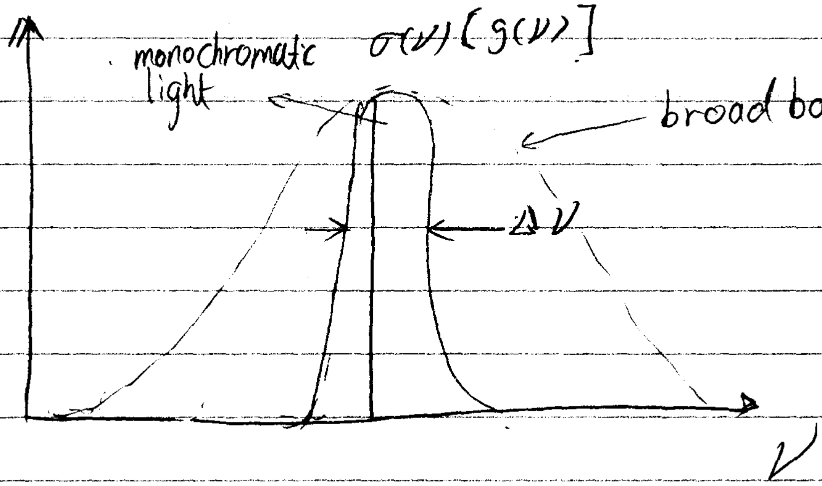
then $$N = \frac{\rho(\nu) V}{h\nu} d\nu$$

$$W = \int \frac{\rho(\nu)}{h\nu} V \frac{c}{V} \sigma(\nu)\, d\nu$$

for narrow transition linewidth

$$W \approx \frac{\rho(\nu_0)}{h\nu_0} c S$$

$$\boxed{W = \frac{\lambda^3}{8\pi h t_{sp}} \rho(\nu_0)}$$



Historically, Einstein defined

spon. trans. prob. $P_{sp} = A$ Einstein coff. of spon. emission

$W = B\rho(\nu_0)$ (broadband)

$\uparrow$

E. coff. of induced emission

Einstein A & B coefficients

So $A = \dfrac{1}{t_{sp}}$. $B = \dfrac{\lambda^3}{8\pi h t_{sp}}$

sometimes still see this notation.

Summary:

$$P_{sp} = \frac{1}{t_{sp}}$$

$$S = \frac{\lambda^2}{8\pi t_{sp}}$$

$$\sigma(\nu) = S\, g(\nu)$$

$$W_{monochromatic} = \frac{\lambda^3}{8\pi h c t_{sp}} g(\nu)\, I$$

$$W_{broadband} = \frac{\lambda^3}{8\pi h t_{sp}} g(\nu_0)\, \rho(\nu_0)$$

$g(\nu)$: source of broadening.

* Lifttime broadening:

For any resonator, linewidth is limited by lifetime of energy in system.

Any excited state has a lifetime Δt

$\Rightarrow$ from uncertainty $\Delta E \cdot \Delta t \geq \frac{h}{2\pi}$

so energy of state has finite width ΔE.