Developing the basic equations we need to study how light & atoms interact.

Important parameter: spontaneous emission lifetime t_{sp}

Summary:

$$P_{spont} = 1/t_{sp}$$

$$\left.\begin{array}{l} S = \lambda^2/8\pi t_{sp} \\ \sigma(\nu) = S\, g(\nu) \end{array}\right\} \text{characterize transition.}$$

stimulated emission / absorbtion

Monochromatic: ν

$$W = \frac{\sigma(\nu)}{h\nu} I$$

$$= \frac{\lambda^3}{8\pi h c t_{sp}} I\, g(\nu)$$

broadband light: (i.e. light from a lamp)

characterize the spectrum of light by energy density

$$\rho(\nu) = \frac{\text{energy}}{(\text{freq. range}) \cdot (\text{volume})}$$

Then # of photons of frequency ν is $\frac{\rho(\nu)}{h\nu} \bar{V} d\nu$

This gives $W = \int \frac{\rho(\nu)}{h\nu} \bar{V} \cdot \frac{c}{\bar{V}} \sigma(\nu) d\nu$

Assuing $\rho(\nu)$ is slowly varying comparing to $\sigma(\nu)$

$$\boxed{W = \frac{\rho(\nu_0)}{h\nu_0} c S} \quad \text{or} \quad = \frac{\lambda^3}{8\pi h t_s} \rho(\nu_0)$$

ν_0 : the center frequency of $\sigma(\nu)$.

These formulas apply to single transition between two states. Usually some degeneracy : sum over states to give transition between levels

Effects : , level 1 g_1-fold , level 2 g_2-fold

i) Each excited state can decay to g_1 lower states.

Give decay rate $\frac{1}{t_s} = \frac{8\pi}{\lambda^2} \bar{S} g_1$

$\bar{S}$: average oscillator strength over different transitions.

ii) Also applies to $W_{2\to1}$.

$$W_{2\to1} = \frac{\bar{\sigma}}{h\nu} I g_1$$

$\bar{\sigma}(\nu)$ = average cross section.

$= \bar{\sigma}\, g(\nu)$

In term of t_s doesn't change:

$$W_{2\to1} = \frac{\lambda^3}{8\pi h c t_s} g(\nu) I.$$

iii) But $W_{1\to2}$ does change

$$W_{1\to2} = \frac{\bar{\sigma}}{h\nu} I g_2 = \frac{\lambda^3}{8\pi h c t_s} \frac{g_2}{g_1} g(\nu) I$$

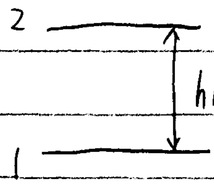
$$\boxed{\frac{W_{1\to2}}{g_2} = \frac{W_{2\to1}}{g_1}}$$

$\sim \quad W_{abs} = W_{stim}$

More about $g(\nu)$: source of broadening.

Lifetime broadening:

consider spontaneous emission from $2 \rightarrow 1$



start in state 2, but decay at rate $1/t_{sp}$

So if P_2 = prob to be in state 2, then

$$\frac{dP_2}{dt} = -\frac{1}{t_{sp}} P_2$$

$$P_2 = e^{-t/t_{sp}}$$

But if atom is in state 2, emits radiation

$$U_{rad} \sim e^{i2\pi\nu_0 t}$$

Intensity of radiation $\propto P_2$,

$$\text{so } I_{rad} \propto e^{-t/t_{sp}}$$

Implies

$$U_{rad} = U_0 e^{-t/2t_{sp}} e^{i2\pi\nu_0 t}$$

Get spectrum from Fourier transform.

$$u(\nu) = \int_{\infty}^{\infty} u_{rad}(t)\, e^{-i2\pi\nu t}$$

$$= u_0 \int_{-\infty}^{\infty} e^{i2\pi(\nu_0-\nu)t - t/t_{sp}}\, dt$$

$$= u_0 \frac{1}{2\pi i(\nu_0-\nu) - \frac{1}{2t_{sp}}}$$

Power spectrum $\quad I(\nu) \sim |u(\nu)|^2$

$$= |u_0|^2 \frac{1}{4\pi^2(\nu_0-\nu)^2 + \frac{1}{4t_{sp}}}$$

FWHM :

$$4\pi^2(\nu-\nu_0)^2 = \frac{1}{4t_{sp}^2}$$

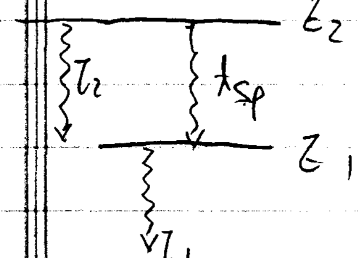
$$\nu-\nu_0 = \pm\frac{1}{4\pi t_{sp}}$$

$$\nu = \nu_0 \pm \frac{1}{4\pi t_{sp}}$$

So $\quad \boxed{\Delta\nu = \frac{1}{2\pi t_{sp}}}$

give normalized $\quad g(\nu) = \dfrac{\Delta\nu/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$

However, in general more than one decay cha channel possible



τ_1, τ_2 = total radiative life times of E_1 & E_2

t_{sp} = life time for decay from E_2 to E_1.

(related to $\sigma_{2\to 1}$, really a measure of coupling strength between two state)

must have $\frac{1}{\tau_2} \geq \frac{1}{t_{sp}}$

Idea: energy of level 2 is uncertain by

$$\Delta E_2 = \frac{h}{2\pi \tau_2}$$

level 1 $\Delta E_1 = \frac{h}{2\pi \tau_1}$

Total frequency uncertainty $\frac{\Delta E_1 + \Delta E_2}{h} = \frac{1}{2\pi}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)$

$$\equiv \frac{1}{2\pi} \frac{1}{\tau}.$$

call τ : transition lifetime. $\tau \leq t_s$

So peak cross section

$$\sigma_0 = S\, g(\nu_0)$$

$$= \frac{\lambda^2}{8\pi t_{sp}} \cdot \frac{2}{\pi \Delta\nu}$$

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$$= \frac{\lambda^2}{8\pi t_{sp}} \cdot \frac{2}{\pi \Delta \nu}$$

$$= \frac{\lambda^2}{4\pi^2 t_{sp}} \cdot 2\pi \tau$$

$$= \frac{\lambda^2}{2\pi} \frac{\tau}{t_s}$$

Maxium possible cross-section $\boxed{\sigma_{max} = \frac{\lambda^2}{2\pi}}$

independent of properties of atom