From last time, to achieve lasing, need

$$G - 1 > \Gamma$$

G: amplification per round trip
Γ: loss per round trip

can write $G = e^{2\gamma_0 l}$
as long as saturation I/I_s doesn't change much during one pass
→ usually requires $\gamma_0 l \ll 1$, often the case

So assume $\gamma_0 l \ll 1$

Then $G \approx 1 + 2\gamma_0 l = 1 + g_0$
$g_0 = 2\gamma_0 l$: small signal gain
(vs. small gain coefficient.)

To lase $g_0 > \Gamma$

Define threshold gain: $g_t = \Gamma = 2\gamma_t l$

and threshold inversion: $\gamma_t = \sigma \Delta N_t$

so $\Delta N_t = \frac{\Gamma}{2 l \sigma}$

Note: $\Delta N_t > 0$ (since $\Gamma > 0$), so always a non-zero threshold even for four-level laser.

Four-level:

$$\Delta N_0 = R\left(\tau_2 - \frac{g_2}{g_1}\tau_1\right) \approx R\tau_2$$

for $\tau_2 \gg \tau_1$

Threshold pumping rate:

$$R_t = \frac{\Delta N_t}{\tau_2} = \frac{\Gamma}{2\ell\sigma\tau_2}$$

required to obtain lasing.

Also, in steady state: gain = loss

$$\text{so} \quad \gamma = \frac{\Gamma}{2\ell} = \frac{\gamma_0}{1 + I/I_s}$$

solve for I:

$$1 + \frac{I}{I_s} = \frac{2\gamma_0 \ell}{\Gamma} = \frac{g_0}{\Gamma}$$

again $g_0 \equiv 2\gamma_0\ell$: small signal gain

$$\boxed{I = I_s\left(\frac{g_0}{\Gamma} - 1\right) = I_s\left(\frac{g_0}{g_t} - 1\right)} = I \text{ in cavity}$$

Recall $\Gamma = 1 - R_1R_2$, for mirrors

add Γ_s for "scattering" losses, which includes all other undesired losses.

$$\Gamma = 1 - R_1R_2 + \Gamma_s$$

But one mirror must serve as output coupler.

say R_1 is output coupler, want $R_1 \neq 1$ to let beam escape.

Say $R_1 = 1 - T_1$ $\quad$ T: transmission

$R_2 = 1 - T_2$

Assume T_2 very small (desired)

$$1 - R_1 R_2 = 1 - (1 - T_1)(1 - T_2)$$
$$= T_1 + T_2 \cancel{+ T_1 T_2}$$

So $\Gamma = T_1 + T_2 + \Gamma_s$

T_1 is useful. $T_2 + \Gamma_s$ are "true" losses

So write $\Gamma = T + L$

$L = T_2 + \Gamma_s$

or generally $= \Sigma$ of all losses besides output coupler.

Then $\boxed{I_{out} = T I_s \left(\frac{g_0}{L + T} - 1 \right)}$

Find optimum T

$$\frac{dI_{out}}{dT} = I_s \left[\frac{g_0}{L+T} - 1 - \frac{g_0 T}{(L+T)^2} \right] = 0$$

so $$g_0 (L+T) - (L+T)^2 - g_0 T = 0$$
$$0 = g_0 L - (L+T)^2$$
$$L+T = \sqrt{g_0 L}$$
$$\boxed{T = \sqrt{g_0 L} - L}$$

At this T,
get $$I_{out} = I_s (\sqrt{g_0} - \sqrt{L})^2$$

Really care about power.
If $g_0 \ll 1$, still have Gaussian beams as usual
Take $I \approx$ average intensity in mode

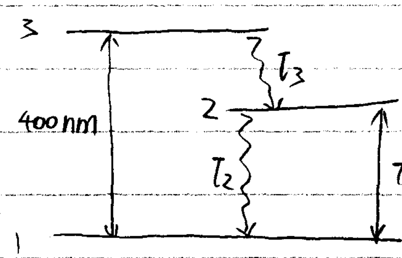
$$P = \pi W^2 I \qquad W: \text{beam waist}$$

so $$P_{out} \approx \pi W^2 I_s T \left(\frac{g_0}{L+T} - 1 \right)$$

W: output beam width
(note, πW^2 factor is heuristic)
I_s: saturation intensity
T: output coupler transmission
L: other cavity losses
g_0: small-signal gain per round trip

Example:

3-level laser (simplified Ruby)



$\tau_2 = 3ms \approx t_{sp}(2\to1)$

$\tau_3 = 50ns$

$t_{sp}(3\to1) = 6\mu s$

$\Delta\nu_{31} = 10^{13}$ Hz

$\Delta\nu_{21} = 3\times10^{9}$ Hz

$n = 1.8$

$N_a = 10^{16}\ cm^{-3}$

Crystal length = 10 cm . cavity length

$T = 0.04, \quad L = 0.02$

If we pump with monochromatic light at 400nm, what is threshold pump intensity?