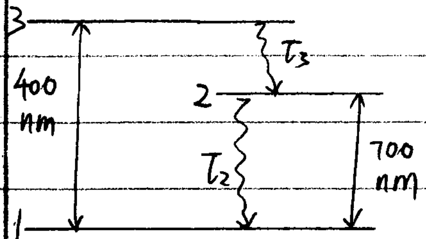
Example: 3-level laser (simplified Ruby)



$\tau_2 = 3ms = t_{sp}(2 \to 1)$

$\tau_3 = 50nm$

$t_{sp}(3 \to 1) = 6\mu s$

$\Delta\nu_{31} = 10^{13}\,Hz$

$\Delta\nu_{21} = 3 \times 10^9\,Hz$

$N_a = 10^{16}\,cm^{-3}$ $\quad n = 1.8$

Crystal length $= 10cm$ = cavity length

$T = 0.04$, $\quad L = 0.02$

If we pump with monochromatic light at 400nm, what is threshold pump intensity?

Threshold gain $\quad g_t = \Gamma = L + T = 0.06$

$g_t = 2\gamma_t l \qquad l = 10cm$

so $\gamma_t = 0.3m^{-1}$

$$\gamma_t \simeq \frac{\lambda^2}{8\pi t_{sp}\Delta\nu}\Delta N_t\Big|_{12}$$

so $$\Delta N_t = \frac{8\pi \times 3ms \times 3\times10^9 Hz \times 0.3m^{-1}}{(700nm/1.8)^2} = 5\times10^{20}m^{-3}$$

From rate equations for three-level system,

$$\Delta N = N_a \frac{\tau_2 W_p - 1}{\tau_2 W_p + 1}$$

So need $\frac{\Delta N_t}{N_a} = \frac{5 \times 10^{20} m^{-3}}{1 \times 10^{22} m^{-3}} = 0.05$

$$(\tau_2 W_p + 1) \frac{\Delta N_t}{N_a} = \tau_2 W_p - 1$$

$$\tau_2 W_p = \frac{1 + \Delta N_t / N_a}{1 - \Delta N_t / N_a} = \frac{1.05}{0.95} = 1.1$$

$$W_p = \frac{1.1}{3 ms} = 370 s^{-1}$$

Finally, $W_p = \frac{\lambda^2}{8\pi t_{sp} \Delta\nu} \frac{I}{h\nu} \Big|_{13}$

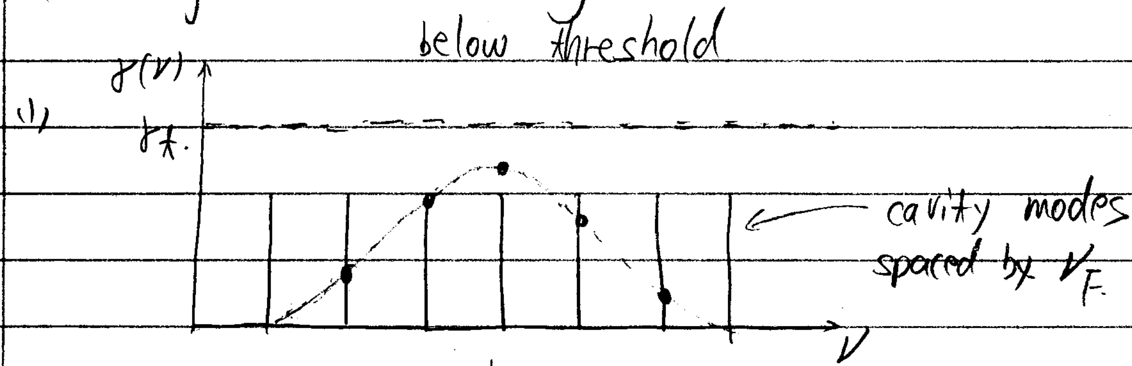
so $I = \frac{8\pi \cdot (6 \mu s) \cdot (10^{13} Hz) \cdot 370 s^{-1} \cdot (6.6 \times 10^{-34} J\text{-}s) \left(\frac{3 \times 10^8 m/s}{400 nm} \right)}{(400 nm / 1.8)^2}$

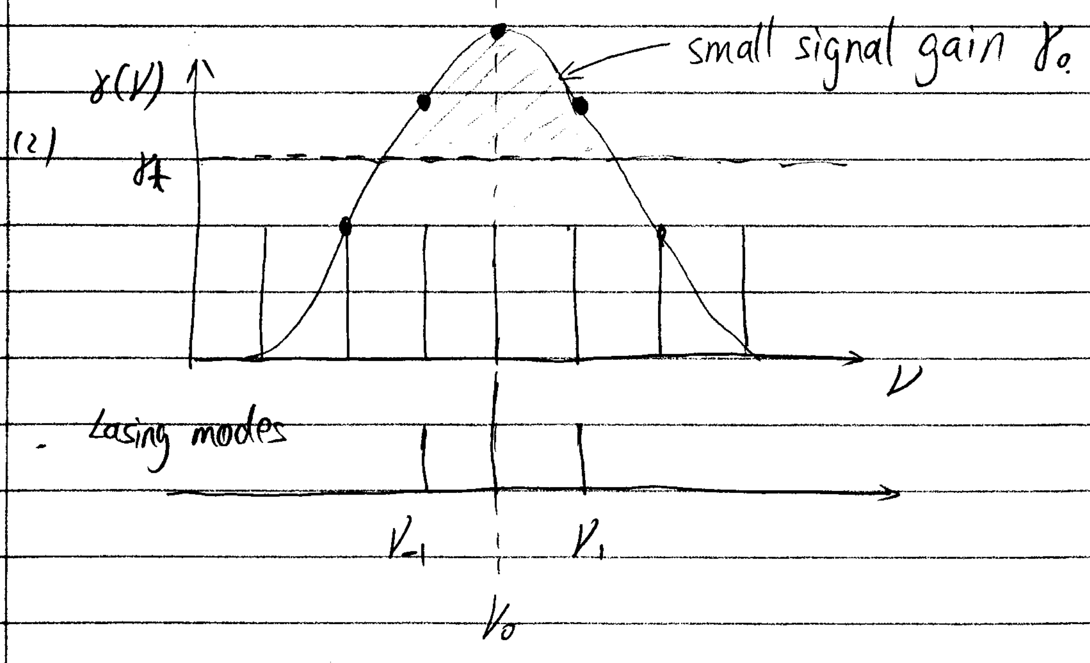
$$\boxed{I_t = 5.6 \times 10^6 W/m^2}$$

Output spectrum

Generally, there are many longitudinal modes of the cavity within the envelop of medium's gain curve. Ideally, we want laser lasing in only one cavity mode. The situations are different for homogeneous and inhomogeneous broadening.

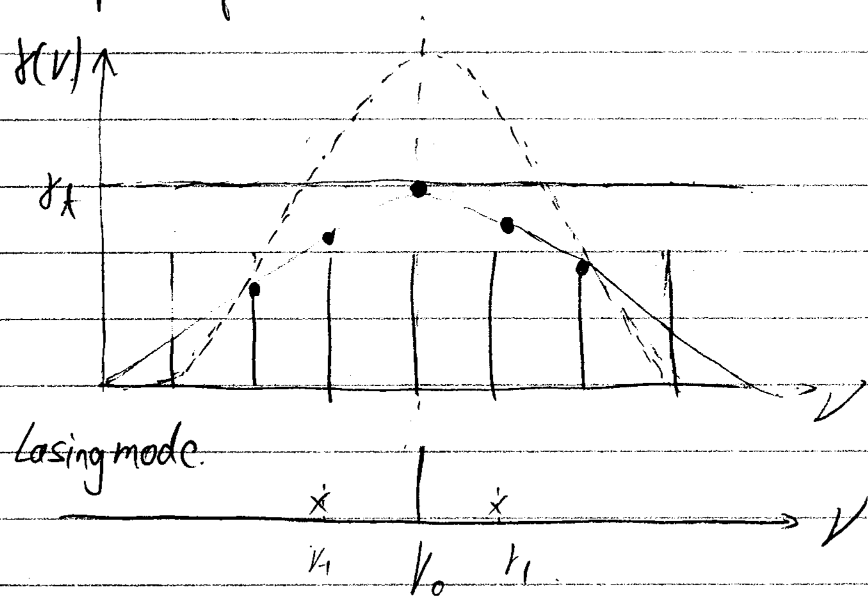
* homogeneous broadening





Immediately following laser turn on, all modes within the gain curve which have gains passing the threshold value, start lasing. in the illustration, these are ν_{-1}, ν_0, ν_1, ν_0 is the central frequency located at the peak of the curve.

(3)



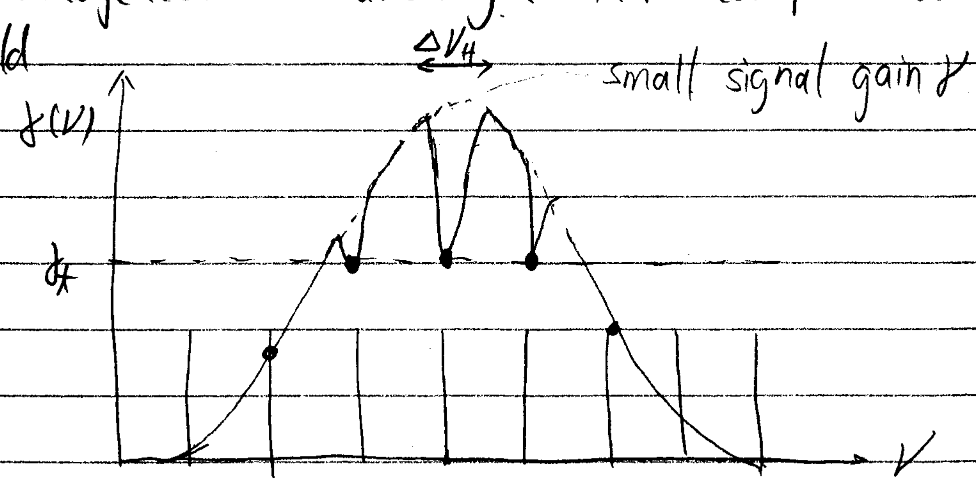
Very short after, the growth of central mode saturates the gain, and the peak of the gain is reduced back to the threshold value. Since the gain curve is homogeneous, the whole curve will be reduced while keeping Lorentzian lineshape. Therefore, for mode ν_{-1} and ν_1, their gain will smaller than loss, and stop lasing.

If the pumping power is higher, the gain envelop will become broader, but the gain of central mode will stay at threshold value, therefore gain of other modes will stay below the threshold.

Only mode with highest gain is able to oscillate in homogeneously broadened gain, so the output is naturally "single mode"

¥ Inhomogeneous broadening: more complicated

Above threshold



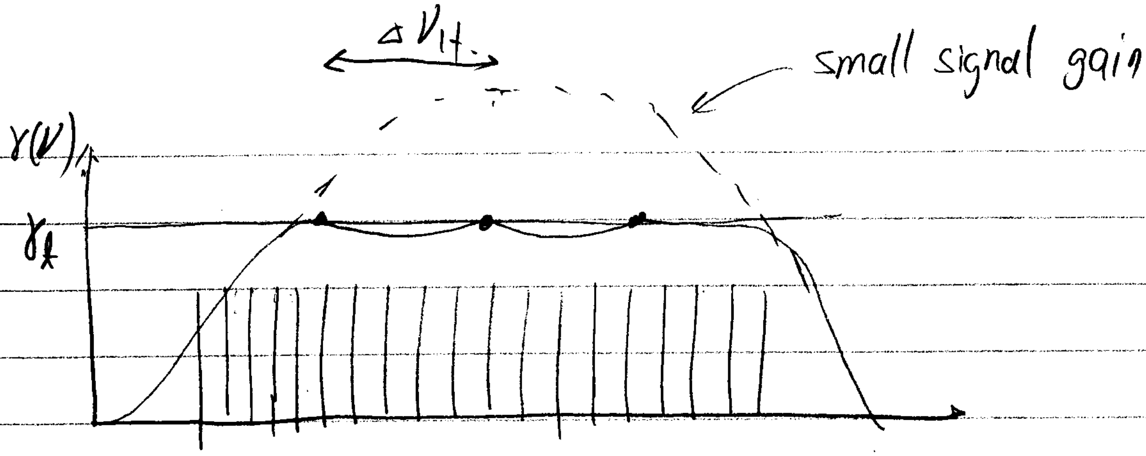
Note: Homogeneous broadening: same for every atom

Inhomogeneous broadening: different for different atoms.
each part on the gain curve from different atoms

If homogenous linewidth is smaller than "mode spacing" ν_F, then each mode interacts with different atoms and saturates independently, there would be a hole at each mode position: "spectral hole burning"

As pump rate increase, more modes lase, output is "multimode"

Often have inhomogeneous broadening, but with homogeneous linewidth large compared to ν_F. The "hole" from different modes will overlap, the whole top part of the gain curve will be "burned"



Then get multimode behavior, but with modes spaced by $\sim \Delta\nu_H$.

Mode hop very easily.

"Spatial hole burning": for homogeneous broadening

For standing wave cavity, there are modes where $I=0$.

So even one mode has saturated, but the atoms at this location are not, and have high gain, so they will support other modes which are displaced from the saturated mode.